

*請在答案卷內作答

參考用

1. (10 points): Let

$$I = \int_C \frac{y}{x^2 + y^2} dx - \frac{x}{x^2 + y^2} dy$$

where C is a circle oriented counterclockwise.(a) Evaluate I if C is given by $(x - 2016)^2 + (y - 2016)^2 = 1$.(b) Evaluate I if C is given by $x^2 + y^2 = 1$.2. (10 points): Find the maximum and minimum values of the function $f(x, y, z) = x^2 - y^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$.

3. (10 points): Compute

$$\lim_{x \rightarrow \infty} \left(\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x} \right)$$

4. (10 points): For what positive x does the following series converge?

$$\sum_{n=1}^{\infty} (\sqrt[n]{x} - 1)$$

5. (10 points): Let $B = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$. Evaluate the integral

$$\iiint_B \frac{x^4 + 2y^4}{x^4 + 4y^4 + z^4} dV.$$

6. (10 points): The n -th derivative of $\frac{1}{x^{2016}-1}$ has the form $\frac{P_n(x)}{(x^{2016}-1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$ for all $n \geq 0$.

7. (20 points): (a) Prove that

$$\int_0^{\infty} \left(\frac{\sin x}{x} \right)^2 dx = \int_0^{\infty} \frac{\sin x}{x} dx.$$

(b) Evaluate the improper integral

$$\int_0^{\infty} \frac{\sin x}{x} dx.$$

8. (10 points): For each continuous function $f: [0, 1] \rightarrow \mathbb{R}$, let $I(f) = \int_0^1 xf(x)(x - f(x)) dx$. Find the maximum value of $I(f)$ over all such functions f .

9. (10 points): Evaluate

$$\int_0^{\infty} \left(x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots \right) \left(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots \right) dx.$$