國立中央大學八十五學年度轉學生入學試題卷

財務管理學系 三年級 科目：統計學 共2頁 第1頁

Instructions: Answer the following questions. Make and state your own assumptions for questions where the information is not sufficient for you to solve them. For example, if you need the corresponding p-value of a normally distributed random variable evaluated at 2.5, you may indicate the value as, for example, $Pr(x \geq 2.5)$, where $x \sim \mathcal{N}(0,1)$.

1. (15 points) Suppose $x_1, ..., x_n \overset{iid}{\sim} \mathcal{N}(\mu_1, \sigma_1^2)$ and $y_1, ..., y_n \overset{iid}{\sim} \mathcal{N}(\mu_2, \sigma_2^2)$ are two independent random samples. Suppose also that the sample estimates for these parameters are: $\bar{x}, \bar{y}, s_x^2, s_y^2$. A statistician suspects that the variance of the first sample is k times the variance of the second sample, where k is a known constant. So he specifies her null and alternative hypotheses: $H_0 : s_x^2 = ks_y^2$ and $H_1 : s_x^2 \neq ks_y^2$. Also, he wants to test if the sample mean of x is twice the sample mean of y, i.e., $H_0 : \bar{x} = 2\bar{y}$ and $H_1 : \bar{x} \neq 2\bar{y}$.

   (a) Are there any problems with his statement?

   (b) How will you test these hypotheses? Please specify the statistics and their distributions under the null. Remember to specify the degrees of freedom, if any.

2. (10 points) Suppose a random variable z is known to have a χ^2 distribution with ν degrees of freedom, and $w = 2z$. Calculate $E(w)$ and $Var(w)$. What do you know about the distribution of w?

3. (10 points) Given a sample of size n, is the $(1-\alpha) \times 100\%$ confidence interval for a parameter always the same as the corresponding acceptance region for that parameter at α significance level? Briefly explain your answer. You may demonstrate by using examples.

4. (15 points) Suppose that on the basis of a random sample of size 250, you are to verify the claim that a population proportion is different from .30.

   (a) If you set the rejection region to be $|\hat{p} - .30| \geq .06$, what is the size of the type I error?

   (b) Determine the numerical value of c so that the test based on the rejection region: $|\hat{p} - .30| \geq c$ has $\alpha = .10$.

5. (20 points) Suppose you are asked to estimate the following model:

$$y_t = b_1 + b_2 x_{2t} + ... + b_k x_{kt} + e_t,$$

   $t = 1, ..., T$. And the OLS estimation gives you: $\hat{b}_1, ..., \hat{b}_k$. However, at the 1%, 5%, and 10% significance levels, none of the estimaates is significantly different from zero except b_1. Therefore, to predict a new observation y_{T+1} with new observation on independent variables x's, the best estimate for y_{T+1} is $\hat{y}_{T+1} = \hat{b}_1$. Give your coments.

6. (10 points) Show that the variable t^2 with v degrees of freedom is a special case of the F variable with $v_1 = 1$ and $v_2 = v$. Here "t" refers to a Student's t variate.

7. (20 points) Suppose you are asked to estimate the following model:

$$STEXP = a_0 + a_1 AID + a_2 W + a_3 STINC + a_4 POP + e$$

where $t = 1, ..., 50$. And you get the following results:

LS // Dependent Variable is STEXP
SMPL range: 1 - 50
Number of observations: 50

VARIABLE	COEFFICIENT	STD. ERROR	T-STAT.	2-TAIL SIG.
C	-89.414484	86.016077	-1.0395090	0.304
AID	4.5006348	0.7642962	5.8886006	0.000
W	-1.3910525	0.8022946	-1.7338425	0.090
STINC	0.0001293	4.222E-05	3.0622413	0.004
POP	-0.5181407	0.1118457	-4.6326405	0.000

R-squared	0.993485	Mean of dependent var	3316.151
Adjusted R-squared	A	S.D. of dependent var	4360.365
S.E. of regression	B	Sum of squared resid	6069400.
Durbin-Watson stat	1.939054	F-statistic	1715.576
Log likelihood	-363.6156		

(a) (10 points) What are A and B?

(b) (10 points The F-statistic given in the table is provided to test "if the model is significant." What are the degrees of freedom for the F-statistic? What is the underlying null hypothesis for this test?