DISCRETE MATHEMATICS

(20%) 1. Find the power sets of each of the following sets:
   (a) $\emptyset$
   (b) $\{\emptyset\}$
   (c) $\{\emptyset, \{\emptyset\}\}$

(20%) 2. In the poset $(P(S), \subseteq)$ of subsets of a set $S$, What is
   (a) $A \land B$?
   (b) $A \lor B$?
   (c) the maximum element?
   (d) the minimum element?

(20%) 3. In a room where there are more than 50 people with ages between 1
   and 100, show that
   (a) Either two people have the same age or there are people whose
       ages are consecutive integers.
   (b) Either two people have the same age or one person's age is a
       multiple of another's.
   (c) Some of the people shake hands. Show that at least two shook
       the same number of hands. ("No hands" is a possibility.)

(20%) 4. A connected graph $G$ has 11 vertices and 53 edges. Show that $G$
   is Hamiltonian but not Eulerian.

(20%) 5. Suppose $T_1$ and $T_2$ are different spanning trees of a connected graph
   $G$, and $e$ is an edge in $T_2$ which is not in $T_1$. Prove that there is an edge
   $f$ in $T_1$ which is not in $T_2$ such that $T_1 \cup \{e\}\{f\}$ is a spanning tree in $G$. 