1. You are visiting an island containing two types of people: knights who always tell the truth and knaves who always lie.

(a) Two natives A and B speak to you as follows:
   A says: B is a knight.
   B says: A and I are of opposite type.
   What are A and B? (5%)

(b) Another two natives C and D approach you but only C speaks.
   C says: Both of us are knaves.
   What are C and D? (5%)

(c) You then encounter natives E and F.
   E says: F is a knight.
   F says: E is a knight.
   What are E and F? (5%)

(d) Finally you meet a group of six natives, U, V, W, X, Y, and Z, who speak to you as follows:
   U says: None of us is a knight.
   V says: At least three of us are knights.
   W says: At most three of us are knights.
   X says: Exactly five of us are knights.
   Y says: Exactly two of us are knights.
   Z says: Exactly one of us is a knight.
   Which are knights and which are knaves. (10%)

2. (a) Does there exist a binary relation R on \{1, 2, 3\} such that R is reflexive, transitive, symmetric, and antisymmetric? Justify your answer. (7%)

(b) Is the union of two equivalence relations always an equivalence relation? Justify your answer. (8%)

(c) If a, b, and c are odd integers, can \(ax^2 + bx + c = 0\) have a rational solution? Justify your answer. (10%)
3. \( K_{m,n,t} \) denotes a complete tripartite graph \( G = (V, E) \) such that

(1) \( V = V_1 \cup V_2 \cup V_3 \), where \( |V_1| = m, |V_2| = n, |V_3| = t \), and \( V_i \cap V_j = \emptyset \) if \( i \neq j \).

(2) There is an edge connecting vertices \( a \) and \( b \) if and only if \( a \in V_i, b \in V_j \) and \( i \neq j \).

(a) Show that \( K_{2,2,2} \) is planar. (7%)

(b) Show that \( K_{3,2,2} \) is nonplanar. (8%)

(c) Find the necessary and sufficient condition in terms of \( m, n, \) and \( t \) such that \( K_{m,n,t} \) is planar. (10%)

4. Develop a general explicit formula for a nonhomogeneous recurrence relation of the form \( a_n = ra_{n-1} + s \), where \( r, s \) and \( a_0 \) are given constants.

(a) \( r = 1 \). (10%)

(b) \( r \neq 1 \). (15%)