類組：物理類　科目：近代物理(2003)　　共 2 頁 第 1 頁

倒扣至本大題(即單選題)0分為止。

一、單選題：答案請填於答案卡。一題五分，答錯倒扣一分，整題不作答不給分也不扣分。

1. The emission through a small hole on the surface of a cavity can simulate the black body radiation. If we assume that the cavity is a two dimensional cavity and everything else is the same, what would be the corresponding Stephan-Boltzman law? (T is the temperature, λ represents the wavelength) (A) total emissive power $\propto T^4$ (B) total emissive power $\propto T^3$ (C) total emissive power $\propto T^2$ (D) the wavelength of the maximum emissive spectrum satisfies $\lambda_{max}T =$ constant (E) the wavelength of the maximum emissive spectrum satisfies $\lambda_{max}T^2 =$ constant.

2. In the Compton scattering, if an x-ray photon whose initial frequency is ν is scattered backwards with scattering angle being $180°$, what would be the kinetic energy of the recoiled electron ? (h =Planck's constant, $m =$ mass of electron, $c =$ speed of light) (A) $\frac{h\nu}{1+\frac{3h\nu}{mc^2}}$ (B) $\frac{h\nu}{1+\frac{2h\nu}{mc^2}}$ (C) $\frac{h\nu}{1+\frac{h\nu}{mc^2}}$ (D) $\frac{h\nu}{1+\frac{mc^2}{2h\nu}}$ (E) $\frac{h\nu}{1+\frac{mc^2}{3h\nu}}$.

3. Consider an ideal gas system composed of helium atoms. Let the density of the gas be ρ and the mass of a helium atom be m. It is known that below the temperature T_D, the system starts to show features of a quantum gas exhibiting properties of identical particles. Which one of the followings is the best estimate of T_D ? (h =Planck's constant, $k_B =$ Boltzmann constant) (A) $\frac{h\rho^{\frac{2}{3}}}{k_Bm}$ (B) $\frac{h\rho^{\frac{1}{3}}}{k_Bm^{\frac{1}{3}}}$ (C) $\frac{h^2\rho^{\frac{1}{3}}}{k_Bm}$ (D) $\frac{h^2\rho^{\frac{2}{3}}}{3k_Bm}$ (E) $\frac{h^2\rho^{\frac{2}{3}}}{3k_Bm^{\frac{5}{3}}}$.

4. The wavefunction of two electrons in an helium atom is given by $\psi(\vec{r}_1,\vec{r}_2)$ with $\vec{r}_1$ and $\vec{r}_2$ being the position vectors of electrons labelled as 1 and 2 respectively. If $\vec{r}_1 \neq \vec{r}_2$, what is the probability density for finding one electron at $\vec{r}_1$ and the other at $\vec{r}_2$? (A) $2|\psi(\vec{r}_1,\vec{r}_2)|^2$ (B) $|\psi(\vec{r}_1,\vec{r}_2)|^2$ (C) $\frac{1}{2}|\psi(\vec{r}_1,\vec{r}_2)|^2$ (D) $\iint |\psi(\vec{r}_1,\vec{r}_2)|^2\, d^3\vec{r}_1 d^3\vec{r}_2$ (E) $\iint 2|\psi(\vec{r}_1,\vec{r}_2)|^2\, d^3\vec{r}_1 d^3\vec{r}_2$.

二、計算題，一題二十分。計算題應詳列計算過程，無計算過程者不予計分

1. A non-relativistic particle of mass m is held in circular orbits around the origin by the force $\vec{F} = -k\vec{r}$, where k is a positive constant and $\vec{r}$ is the position vector of the particle.
(a) (10%) By using the Bohr quantization rule and taking $r = \infty$ as the zero point for the potential energy, find allowed square of distances r^2 and total energies of the particle.
(b) (5%) Find the uncertainty relation between the uncertainty of position Δr and the uncertainty of momentum Δp.
(c) (5%) By using uncertainty relations, estimate the ground state energy of the particle.

2. A K^+ meson at rest can decay into pions through the channel: $K^+ \rightarrow \pi^+ + \pi^0$. Following the decay, the π^+ meson will further decay into a μ^+ meson and a neutrino. Let rest masses of K^+、π^+、π^0 and μ^+ meson be m_K、m_{π^+}、m_{π^0} and m_{μ^+}, and

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ignore the rest mass of the neutrino. Denote $\gamma_{\pi^+} = 1/\sqrt{1-(v_{\pi^+}/c)^2}$ and $\gamma^*_{\mu^+} = 1/\sqrt{1-(v_{\mu^+}/c)^2}$, where v_{π^+} is the speed of π^+ in the rest frame of K^+ and where v_{μ^+} is the speed of μ^+ in the rest frame of π^+. Answer the following questions (express your answers in terms of m_K、m_{π^+}、m_{π^0}、m_{μ^+}、γ_{π^+} and $\gamma^*_{\mu^+}$):

(a) (7%) Find the energy of π^+ in the rest frame of K^+.

(b) (3%) Find the energy of μ^+ in the rest frame of π^+.

(c) (10%) Find the maximum and minimum energies of μ^+ in the rest frame of K^+

3. Consider a particle of mass m and energy E incident from $x<0$ into the potential barrier: $V(x)=W>0$ for $0\le x\le L$ and $V(x)=0$ otherwise. For $E=\frac{\hbar^2k^2}{2m}$, the wavefunction of the particle can be written as $\psi(x)=e^{ikx}+re^{-ikx}$ for $x<0$ and $\psi(x)=te^{ikx}$ for $x>L$. Answer the following questions:

(a) (8%) Let $E<W$ and $q=\sqrt{2m(W-E)}/\hbar$, $\psi(x)$ can be generally written as $Ae^{-qx}+Be^{qx}$ for $0\le x\le L$. From boundary conditions of $\psi(x)$, find equations that $r, t, A,$ and B satisfy.

(b) (5%) For $qL\gg 1$, the transition coefficient T is approximately by $T=f(E,W)e^{-2qL}$. Find $f(E,W)$.

(c) (7%) Consider the fusion of two deuterons due to quantum tunneling. The mass of a deuteron is m and the potential $V(r)$ between deuterons is $V(r)=0$ for $r\le a$, and $V(r)=e^2/r$ for $r>a$. Here r is the distance between two deuterons and fusion occurs when $r\le a$. Before head-on collision, the kinetic energy of each deuteron is E and $E=2e^2/\mathrm{b}$. By using results of problem (b) and ignoring $f(E,W)$, estimate the probability of fusion for each head-collision of deuterons.

4. Consider 7 non-relativistic identical particles in a three dimensional box of size $L\times 2L\times$2L. such that the potential that acts each particle is given by $V(x,y,z)=0$ for $0\le x\le L$, at the same time, $0\le y\le 2L$ and $0\le z\le 2L$; and $V(x,y,z)=\infty$ otherwise. Here (x,y,z) is the Cartesian coordinate of the particle. Suppose that the mass of each particle is m and assume that there is no interaction among particles, answer the following questions.

(a) (12%) Suppose that the spin of these particles is $1/2$. Find total energies and the corresponding degeneracy of the system for the ground state, the 1st excited state and the 2nd excited state.

(b) (8%) Suppose that the spin of these particles is 0. Find total energies of the system for the ground state, the 1st excited state and the 2nd excited state. Find the degeneracy of the 2nd excited state.

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