科目：應用數學(2001)

校系所組：中央大學光電科學與工程學系照明與顯示科技碩士班
交通大學電子物理學系（丙組）
交通大學物理研究所
清華大學物理學系
清華大學先進光源科技學位學程（物理組）
清華大學材料科學工程學系（乙組）
陽明大學生醫光電研究所（理工組）
清華大學天文研究所

參考用

一. (15%) Consider a linear mapping $T$ that acts on the linear vector space $V$ formed by all $2\times 2$ real matrices. For any matrix $A$ that belongs to $V$

$$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

one finds that $T[A]$ is given by

$$T[A]=\begin{pmatrix} 2c & a+c \\ b-2c & d \end{pmatrix}.$$

Find all independent $\lambda$ and matrices $A$ with $a,b,c,d$ being relatively prime integers such that $T[A]=\lambda A$ is satisfied.

二. (8 %) Evaluate the integral $\iint \vec{r}\cdot\hat{n}\,ds$ over the surface (which is a triangle) in the first octant formed by the plane $2x+3y+5z=30$ and x, y, z axes. Here $\hat{n}$ is the unit normal vector to the surface, pointing away from the origin.

三. (12%) Consider a vector field $\vec{V}(x,y,z)=(2xy+y^2,2xy+x^2,z)$.

(a) Find the line integral $\int_C \vec{V}\cdot d\vec{r}$ along the path C from (0,0,0) to (1,1,1) for C being the curve intersecting by two surfaces: $y-x^2=0$ and $z^3-x=0$.

(b) Consider the same line integral from (0,0,0) to (1,1,1) but with C being from (0,0,0) to (1,0,0) and then from (1,0,0) to (1,1,0) and finally to (1,1,1). All intermediate connections are straight lines. Is the value of the line integral the same? Why? Is $\vec{V}$ conservative? If your answer is yes, construct its potential function.

四. (15%) Consider a matrix

$$A=\begin{pmatrix} -2 & -1 \\ 4 & 3 \end{pmatrix}.$$

Let $B=A+A^2+A^3+.....+A^{10}$. Find explicit expression of $B$ as a $2\times 2$ matrix.

注意：背面有試題

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五. (15%) Find the Laurent series for the function $\dfrac{1}{(z-1)(z-2)}$ in each of the following domains:

(a) $|z|<1$, (b) $1<|z|<2$, (c) $|z|>2$.

六. (15%) Perform the integral $\int_0^\infty \dfrac{\sqrt{x}}{x^2+1}dx$ by contour integral method.

七. (20%) Solve the following PDE boundary problem.

$$\begin{cases} u_t = c^2 u_{xx} \quad (0<x<l) \\ u(t,0) = u(t,l) = 0 \\ u(0,x) = x(l-x) \end{cases}$$