類組：電機類　科目：工程數學D(3006)　

※請在答案卷內作答

一 (20分，計算題) Let

$$A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & -1 & -1 \\ 0 & 1 & 2 & 1 \end{bmatrix}.$$

（一） (5分) Find the dimension of Null A, where Null A is the null space of A.

（二） (10分) Let W be a subspace of $\mathcal{R}^4$ that contains every vector that is in both Null A and Null B. Find a basis for W.

（三） (5分) Let U be a subspace of $\mathcal{R}^4$ that contains every vector that is in both the column space of A^T and the column space of B^T. Find a basis for U.

參考用

二 (30分，計算題) Let

$$A = \begin{bmatrix} 1 & 0 & -1 & -1 \\ -1 & 1 & 2 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}.$$

Suppose P is a 4×4 matrix such that $P\mathbf{x}$ is the orthogonal projection of $\mathbf{x}$ on Null A for every $\mathbf{x}$ in $\mathcal{R}^4$.

（一） (5分) Is P unique? Explain your answer. (No credits for answers without justifications.)

（二） (5分) Find the eigen values of P.

（三） (10分) Find the orthogonal projection of $\mathbf{b}$ on Null A.

（四） (5%) What is the distance between $\mathbf{b}$ and the orthogonal complement of Null A?

（五） (5%) Find a solution of $\mathbf{x}$ that minimizes $||A^T\mathbf{x} - \mathbf{b}||$.

注意:背面有試題

※請在答案卷內作答

三 (30分，計算題)

(一) (6分) Solve the differential equation

$y'(t)+(\frac{3-2t^2}{t})y(t)=\frac{-10}{t^2}$.

(二) (6分) Solve the differential equation

$(4xy+3x^2)dx+(2y+2x^2)dy=0$, $y(0)=-2$.

(三) (4分) Consider the initial value problem:

$y''(t)+p_0y'(t)+q_0y(t)=f(t)$, $t\geq 0$, $y(0)=0$, $y'(0)=0$.

Determine the constants p_0 and q_0 so that the solution $y(t)$ can be expressed as $y(t)=\int_0^t e^{-2(t-\tau)}sin(t-\tau)f(\tau)d\tau$.

(四) (8分) Solve the initial value problem :

$y''(t)+y(t)=\sum_{k=1}^{\infty}\delta(t-k\pi)$, $y(0)=0$, $y'(0)=1$,

and plot $y(t)$ on the interval $[0,4\pi]$.

(五) (6分) Determine the eigenvalues and the associated eigenfunctions of the Sturm-Liouville problem:

$y''(t)+\lambda y(t)=0$ $(0<x<L)$; $y'(0)=0$, $y'(L)=0$.

注意:背面有試題

※請在答案卷內作答

四 (20分，計算題)

(一) (10分) Solve the initial value problem :

$$y''(t) + 9y(t) = cos(3t) + 2sec(3t),\ y(1) = 0,\ y'(0) = -1.$$

(二) (10分) Let $A = \begin{bmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$. It is known that A has an eigenvalue $\lambda = -1$ of multiplicity 3 with only one linearly independent eigenvector $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Let $\vec{u} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

It is also known that $\vec{v}$, $\vec{u}$ and $\vec{w}$ satisfy the following equations:

$$(A + I)\vec{u} = \vec{v}$$
$$(A + I)\vec{w} = \vec{u}$$

and $e^{At} = \begin{bmatrix} -2t+1 & 0 & K_2 \\ K_1 & 1 & t^2 - t \\ t & 0 & K_3 \end{bmatrix} e^{-t}$. What are K_1, K_2 and K_3?