科目：工程數學A(3003)

校系所組：中央大學電機工程學系(電子組、系統與生醫組)
交通大學電子研究所(甲組、乙A組、乙B組)
交通大學電控工程研究所(甲組)
交通大學生醫工程研究所(乙組)
清華大學電機工程學系(甲組)
清華大學光電工程研究所
清華大學電子工程研究所
清華大學工程與系統科學系（丁組）
陽明大學生物醫學工程學系（醫學電子組）

1. (15%) Let V be the set of all 2-by-2 real symmetric matrices, i.e.,

$$\mathsf{V}=\left\{\begin{pmatrix} a & b \\ b & c \end{pmatrix} : a,\ b,\ c\in\mathbb{R},\ \text{where } \mathbb{R} \text{ the set of real number}\right\}$$

and let W be the set of all 2-by-2 real matrices. Let $\mathsf{T}:\mathsf{V}\rightarrow\mathsf{W}$ be a linear transformation defined by

$$\mathsf{T}\left(\begin{pmatrix} a & b \\ b & c \end{pmatrix}\right)=\begin{pmatrix} a-c & -b+c \\ -2a+b+c & -a+c \end{pmatrix}.$$

(a) (5%) Find a basis γ for the range space of T. (Please show details of your derivations.)

(b) (5%) Consider the Frobenius inner product where the inner product between

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \text{ and } B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \in \mathsf{W}$$

is defined by $< A, B >= a_{11}b_{11}+a_{12}b_{12}+a_{21}b_{21}+a_{22}b_{22}$.

Find an orthonormal basis α for the range space of T using the Gram-Schmidt process on the basis γ obtained in (a). (Please show details of your derivations.)

(c) (5%) Let

$$\beta=\left\{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right\}$$

be an ordered basis for V and let α be the orthonormal basis obtained in (b). Find the matrix representation of T in the ordered basis β and α, i.e., $[\mathsf{T}]_{\beta}^{\alpha}$.

2. (10%) Suppose that

$$\det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}=-5$$

and $b_1c_2=-b_2c_1=1$. Compute the determinant of the matrix M. Please show your derivations.

$$M=\begin{pmatrix} b_1+2c_1 & b_2+2c_2 & b_3+2c_3 \\ a_1 & a_2 & a_3+2 \\ 3b_1+c_1 & 3b_2+c_2 & 3b_3+c_3 \end{pmatrix}$$

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參考用

3. (25%) In the s-domain, via Laplace transform, one has

$$Y(s)=\frac{1}{s^2+16}+\frac{s}{(s^2+16)^2}-\frac{s}{(s^2+16)^2}e^{-\pi s}$$

(a) (9%) If this $Y(s)$ is used to describe a 2nd-order Ordinary Differential Equation:

$$y''(t)+p(t)\,y'(t)+q(t)\,y(t)=r(t),$$

with the initial conditions: $y(0)=0$ and $y'(0)=1$, find $p(t)$, $q(t)$ and $r(t)$, respectively.

(b) (5%) Plot $r(t)$.

(c) (5%) Find the solution $y(t)$.

(d) (6%) Plot $y(t)$.

4. (15%) Assume $x \neq 0$, for a 2nd-order differential equation: $x^2y''+xy'+y=4\sin(\ln x)$.

(a) (6%) Please find the general solution of the corresponding homogeneous ODE in real function format.

(b) (9%) Please find the particular solution that satisfies the given non-homogeneous ODE.

5. (15%) Legendre equation meets the Sturm-Liouville problem on the interval: $-1 \leqq x \leqq 1$ and the solution can be expressed as a generalized Legendre-Fourier series:

$$y(x)=\sum_{n=0}^{\infty}a_m y_m(x)$$

where $y_m(x)$ is the corresponding eigenfunction of the Legendre equation, then

$$\int_{-1}^{1}y_m(x)y_n(x)dx=\delta_{mn}=\begin{cases}1 & \text{if } m=n\\ 0 & \phantom{\text{if }} m\neq n\end{cases}$$

If $y(x)=x^2+2x+3$ is a solution for this Legendre equation, please express $y(x)$ as a generalized Legendre-Fourier series. (Please show details of your derivations.)

6. (20%) For a complex function $f(z)=(1+z^3)^{-1}$,

(a) (6%) Please find the residues of $f(z)$.

(b) (14%) Applying the results in (a) to evaluate the integral by contour integration in the complex plane

$$I=\int_0^{\infty}\frac{dx}{1+x^3}$$