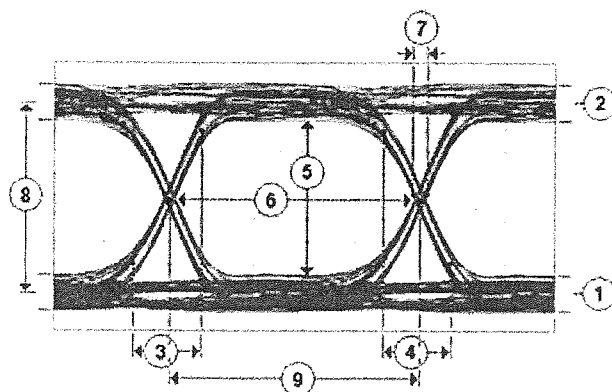


Note: Detailed derivations are required to obtain a full score for each problem.

1. (14%) In baseband communication systems, an eye diagram/pattern is often used to analyze the quality of an on-off keying (OOK) signal. A typical eye diagram is given in the diagram below:



Please answer the following questions. (Note: A short description that justifies your answer is required to get full credit for each problem.)

- What are the possible line codes for this diagram? (A) unipolar NRZ, (B) bipolar NRZ, (C) unipolar RZ and (D) bipolar RZ. (3%)
 - From measurements ① to ⑨ in the diagram, which one can be used to estimate data rate? (3%)
 - From measurements ① to ⑨ in the diagram, please describe how you can estimate the error probability. (8%)
2. (16%) The power of signal $s(t)$ is defined as

$$P_s = \lim_{T \rightarrow \infty} \left\{ \frac{1}{2T} \int_{-T}^T |s(t)|^2 dt \right\}.$$

(That is, the signal $s(t)$, in voltage, is applied across a resistor with 1 ohm.)

- Suppose that $s(t)$ is an amplitude modulated (AM) signal given by

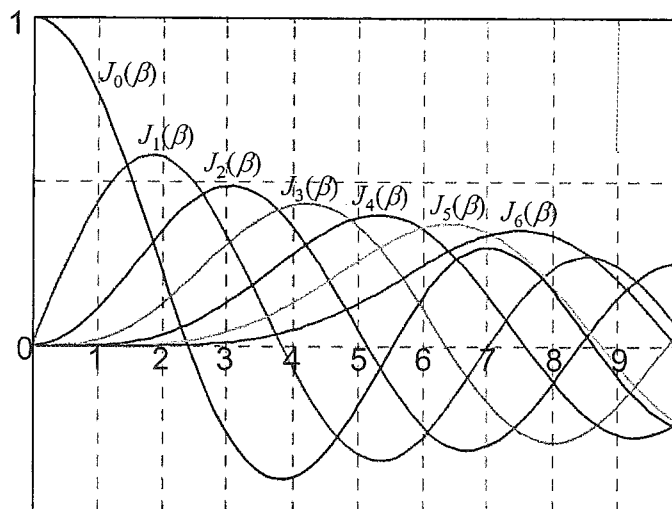
$$s(t) = A_c(a_0 + m(t)) \cos(2\pi f_c t),$$

where $m(t) = A_m \cos(2\pi f_m t)$ is the baseband signal with amplitude A_m and frequency f_m , and A_c and f_c are the carrier's amplitude and frequency, respectively. If there is no over-modulation for all time t , what is the maximum possible value of P_s ? (4%)

- (b) Following (a), assume that the carrier amplitude, A_c , is 2 volts and the message $m(t)$ has a frequency $f_m = 10$ KHz and a power equal to 2 watts. Moreover, assume that the total signal power, P_s , is 22 watts. Is this an over-modulated signal? Why? (4%)
- (c) Repeat (a), for a frequency modulated (FM) signal given by

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t],$$

where k_f is the frequency sensitivity and β is the phase deviation. An interesting behavior of FM is that, there exists some specific frequency deviation, Δf , such that the carrier component is totally suppressed, i.e., no carrier component. According to the following plot of the n th order Bessel function of the first kind, please use Carson's rule to estimate the smallest bandwidth, in terms of f_m , such that there is no carrier component in the transmitted FM signal $s(t)$. (4%)



- (d) At the receiver, we first pass the ideal FM signal $s(t)$, as shown in (c), through an ideal bandpass filter (BPF) with a bandwidth equal to what you obtained in (c). Then, we demodulate this filtered FM signal with an ideal FM demodulator. Can we get an undistorted $m(t)$ at the output of demodulator? Why? (4%)

3. (25%) Let us consider a sinusoidal wave with random phase given by

$$X(t) = A \cos(2\pi f_c t + \Theta),$$

where A and f_c are known constants and Θ is a random variable that is uniformly distributed over the interval $[-\pi, \pi]$.

- (a) Please derive the autocorrelation function of $X(t)$. (5%)
- (b) Please derive the power spectral density (PSD) of $X(t)$. (5%)
- (c) Please derive the PSD of $Y(t) = Z(t)X(t)$, where $Z(t)$ is a stationary process that is independent of Θ . Please express the result in terms of the PSD of $Z(t)$. (5%)
- (d) Please derive the PSD of $X(t)$ if f_c is no longer a constant value but a random variable which is independent of Θ . (10%)

4. (10%) In the $(k+1, k)$ single-parity-check (SPC) code, a single parity bit b_1 is appended to a block of k message bits (m_1, m_2, \dots, m_k) . The single parity bit, b_1 , is chosen so that the codeword satisfies the even parity rule:

$$m_1 \oplus m_2 \oplus \dots \oplus m_k \oplus b_1 = 0,$$

where \oplus represents the binary addition.

- (a) Please give a k -by- $(k+1)$ generator matrix \mathbf{G} in systematic form for the $(k+1, k)$ SPC code. (5%)
- (b) Please give a 1 -by- $(k+1)$ matrix \mathbf{H} such that $\mathbf{H}\mathbf{G}^T = 0$. (5%)

5. (20%) Consider M signals

$$s_i(t) = A \cos(2\pi f_i t + \theta_i), \quad 0 \leq t < T_s, \quad i = 1, 2, \dots, M,$$

where θ_i represents the carrier phase at the initial time, $f_i \gg 0$, and M is a power of 2.

- (a) Derive the correlation of the two signals $s_i(t)$ and $s_j(t)$ over the interval $(0, T_s)$. (5%)
- (b) Determine the minimum spacing Δf between f_i and f_j that maintains the orthogonality between the two signals $s_i(t)$ and $s_j(t)$ when $\theta_i = \theta_j$. (2%)
- (c) Determine the minimum spacing Δf between f_i and f_j that maintains the orthogonality between the two signals $s_i(t)$ and $s_j(t)$ for any random phases $\theta_i \neq \theta_j$. (3%)
- (d) Assume that a transmitter forwards a data stream to a receiver through an additive white Gaussian noise (AWGN) channel by using the set of M orthogonal signals described above. The transmit power is $P = 0$ dBW, the transmission bandwidth is $B = 4.5$ MHz, the propagation loss of the channel is $L = 40$ dB, and the noise power spectral density is $N_0 = -110$ dBW/Hz. If the required energy per symbol to noise power spectral density ratio at the receiver is $E_s/N_0 \geq 10$ dB, find the maximum achievable transmission rate (in bits per second) for the cases $\theta_i = \theta_j$ and $\theta_i \neq \theta_j$, respectively. (5%)

- (e) Following (d), if the propagation loss of the channel is increased to $L = 46$ dB, find the maximum achievable transmission rate for the cases $\theta_i = \theta_j$ and $\theta_i \neq \theta_j$, respectively. (5%) [Hint: $\log_{10} 2 \approx 0.3$]
6. (15%) Consider the design of a differential 8-PSK modulation scheme, which is similar to $\pi/4$ -Differential QPSK modulation.
- (a) Illustrate the two constellations, including the phases of all signal points, alternately used in two successive symbols. (4%)
- (b) Determine the mapping of Gray-encoded input data with respect to the phase change $\Delta\theta$, where "000" has the minimum positive phase change and the signal point "000" is situated between the two points "100" and "001". (4%)
- (c) According to (b), if the initial carrier phase is $\theta_0 = 0$, find the transmitted phase θ_k for the following input data sequence "1 1 1 1 0 0 1 0 1 0 1 0 0 1 0 0 0 1". (3%)
- (d) If a rectangular pulse shaping waveform is used and the maximum transmitted signal amplitude is \sqrt{E} , determine the minimum envelope amplitude of all possible modulation trajectories. (4%) [Hint: Express the answer by using a trigonometric function.]