

※請在答案卷內作答

1. (20 %) Short Quiz. **Note:** You'll get 0 point if your answer doesn't include any explanation.
- (a) (6 %) QAM is a signal that transmits two independent sets of information on a radio frequency carrier but with quadrature phase offset. Can we transmit such a signal without up-converting these two sets signal to a radio frequency? Why?
- (b) (8 %) To demodulate an FM signal: $s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau)$, where A_c, f_c, k_f and $m(t)$ are carrier amplitude, carrier frequency, frequency sensitivity and information, respectively, we can use a frequency discriminator which consists of a slope circuit followed by an envelope detector. Please explain the operation principle of this configuration.
- (c) (6 %) For a unipolar NRZ signal, bit "1" carries A volt and bit "0" carries 0 volt. In practice, we usually find that the noise accumulated in bit "1" is typically larger than it in bit "0" at the receiver. How to explain this phenomenon in terms of noise types in the receiver's circuits?

2. (20 %) A random process, defined by $X(t) = A(t) \cos(2\pi f_c t + \Theta)$, is applied to an integrator which produces an output of $Y(t)$ as $Y(t) = \int_{t-T}^t X(\tau) d\tau$.

- (a) (5 %) Suppose that the carrier frequency f_c is a constant, $A(t)$ is a WSS random process which is independent of Θ , and Θ is a random variable uniformly distributed in $[0, 2\pi]$. Please find the power spectral density, $S_X(f)$, of $X(t)$ in terms of the power spectral density, $S_A(f)$, of $A(t)$.
- (b) (5 %) Please find and plot the corresponding impulse response of the employed integrator. **Note:** You'll get 0 point if you only give the answer without any justification.
- (c) (5 %) Please find the power spectral density, $S_Y(f)$, of output $Y(t)$.
- (d) (5 %) If $A(t)$ is a sequence of random binary bits with amplitude A and bit time T_B whose autocorrelation function can be defined as

$$R_A(\tau) = \begin{cases} A^2 \left(1 - \frac{\tau}{T_B}\right) & |\tau| \leq T_B \\ 0, & |\tau| > T_B \end{cases},$$

please find the power spectral density, $S_Y(f)$, of the output $Y(t)$.

3. (10%) A signal is decomposed into a complete set of orthonormal basis, i.e. $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$, represented as $s(t) = s_1\phi_1(t) + s_2\phi_2(t) + s_3\phi_3(t)$. Compute the energy of $s(t)$, expressed by $\{s_1, s_2, s_3\}$.

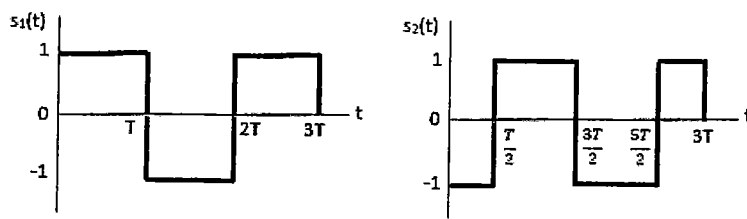
注意：背面有試題

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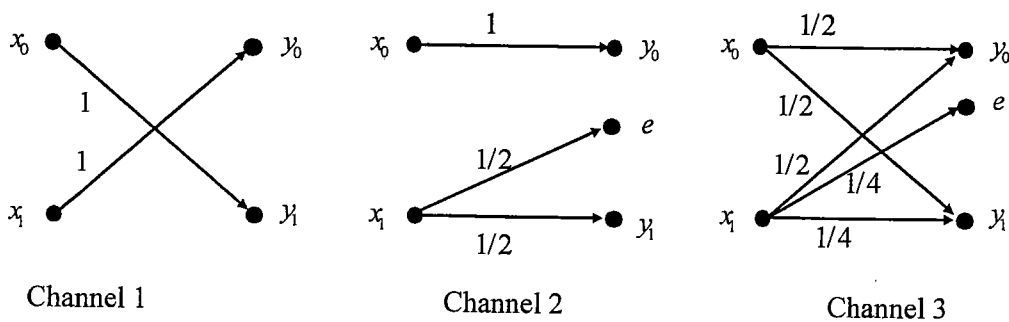
4. (10%)

- (a) (5%) For a signal set $\{s_1, s_2, \dots, s_{32}\}$ with equal probability, a receiver receives the Tx signal correctly. How much bits of information can be represented for each symbol from information theory perspective?
- (b) (5%) For a signal set $\{s_1, s_2, \dots, s_{20}\}$ with equal probability, a receiver receives the Tx signal correctly. How much bits of information can be represented for each symbol from information theory perspective?

5. (20%) The following figure shows a pair of signals $s_1(t)$ and $s_2(t)$ that are orthogonal to each other over the observation interval $0 \leq t \leq 3T$. The received signal is defined by $X(t) = s_k(t) + w(t)$, $k=1,2$, where $w(t)$ is white Gaussian noise of zero mean and PSD $N_0/2$. Please calculate the average probability of symbol error (expressed in error function or complementary error function is sufficient) for $E/N_0=4$, where E is the signal energy.



6. (20%) Consider transmission of information source X with $p(X = x_0) = p$ and $p(X = x_1) = 1 - p$ over the discrete memoryless channels 1, 2, and 3 with transition probabilities as the following figure. The output of the channel is represented by $Y = y_0, y_1$, or e , in which $Y = e$ denotes that the information is erased by the channel.



- (a) (6%) For Channel 1, please find the input distribution probability p that maximizes the mutual information $I(X;Y)$, i.e. the capacity.
- (b) (6%) For Channel 2, please find the input distribution probability p that maximizes the mutual information $I(X;Y)$, i.e. the capacity.
- (c) (8%) Let C_1, C_2 and C_3 denote the capacity of channels 1, 2, and 3, respectively. Please compare the capacity of C_3 with $(C_1+C_2)/2$. Which is larger and why?