類組：電機類　科目：通訊系統(通訊原理)(300E)　　　　共 3 頁 第 1 頁

※請在答案卷內作答

*** Note: You must give detailed derivations, otherwise you get no points.**

For your information:

- The probability density function of a multivariate Gaussian random vector $\mathbf{X} = [X_1, \ldots, X_n]^T$ is

$$f_{X_1,\ldots,X_n}(x_1,\ldots,x_n) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{(\mathbf{X}-\mu)^T \Sigma^{-1} (\mathbf{X}-\mu)}{2}\right)$$

  where the superscript T denotes transposition; μ is the mean vector; Σ is the covariance matrix, and $|\Sigma|$ is the determinant of covariance matrix Σ.

- The Q-function is defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2} du$

一、(25%) In a pulse-code modulation (PCM) system, 20 analog channels, with a bandwidth 15 kHz per channel, are time-division multiplexed for transmission. The number of representation levels used in a uniform quantizer is 64.

(一)、 (5%) If the sampling rate is the Nyquist rate, determine the overall transmission rate (in bits per second, bps) of the PCM system.

(二)、 (5%) If the baseband M-ary transmission is applied in the encoder, determine the overall symbol rates (in symbols per second, sps) of the PCM system for M = 4, 8 and 16.

(三)、 (5%) Draw the signal waveform (amplitude vs. symbol) of the data sequence "101010100111000110010011" for 8-ary PAM transmission, where the symbol representation is based on Gray encoding and the symbols '000' and '100' are represented, respectively, as the highest and lowest levels.

(四)、 (5%) If the analog signal is a sinusoidal function, find the output signal-to-noise power ratio (in dB) of the PCM system.

(五)、 (5%) According to (四), if the minimum acceptable output signal-to-noise power ratio is changed to 45 dB, determine the minimum overall transmission rate (in bps) of the PCM system.

[Hint: (1) The quantization noise power is $\sigma_Q^2 = \Delta^2/12$, where Δ is the step-size of the quantizer. (2) $\log_{10} 2 \approx 0.3$, $\log_{10}(3/2) \approx 0.18$.]

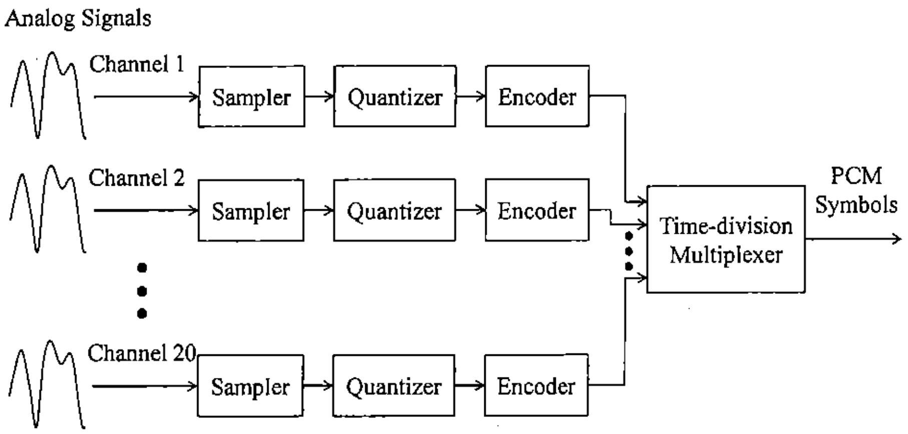

Fig. 1. The considered pulse-code modulation (PCM) system.

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二、(25%) An Armstrong indirect frequency modulation (FM) modulator is depicted as Fig. 2. The design is to generate an FM signal with carrier at f_{c4} = 97.3 MHz and Δf_4 = 10.24 kHz, where Δf denotes the peak frequency deviation. A narrow band FM (NBFM) signal generator generates a signal with f_{c1} = 20 kHz and Δf_1 = 5 Hz. Assume both the frequency multipliers multiply the input frequencies by 2^{M_1} and 2^{M_2} times. The local oscillator can generate a sinusoidal wave from 400 kHz to 500 kHz for frequency mixing.

(一)、 (5%) How many total multiples should be provided by the two frequency multipliers?

(二)、 (5%) Find a relation among the following frequencies: f_{c1}, f_{c4}, and f_{LO}.

(三)、 (15%) According to the design goal, find the optimized specifications of 2^{M_1}, 2^{M_2} and f_{LO}.

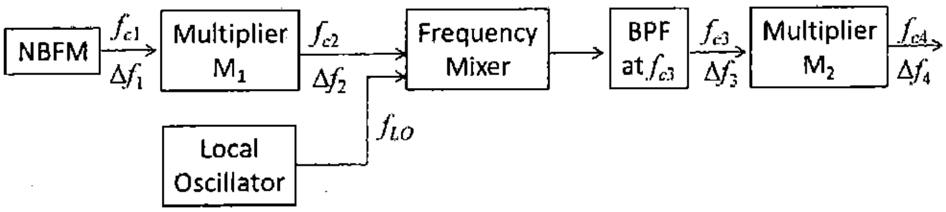

Fig. 2 Armstrong FM generator

三、(12%) A communication system of transmit antipodal symbol s_m (i.e. $s_1 = +\sqrt{E_b}$ and $s_2 = -\sqrt{E_b}$) with equal probability over the fading channel can be represented as

$$Y = \rho s_m + W, \qquad m = 1, 2,$$

where the coefficient ρ is a random variable denoting the channel fading, and W is an additive white Gaussian noise with zero mean and variance $N_0/2$.

(一)、 (4%) Assuming that $\rho = 0.5$ with probability of 1, please find the maximum likelihood decision rule for the receiver and the resulting bit error probability.

(二)、 (4%) Assuming that ρ takes values of +1 or −1 with equal probability, please find the maximum likelihood decision rule for the receiver and the resulting bit error probability.

(三)、 (4%) Assuming that ρ takes values of +1 or 0 with equal probability, please find the maximum likelihood decision rule for the receiver and the resulting bit error probability.

四、(13%) Consider a communication system with the M-ary FSK signaling $s_i(t) = \sqrt{\frac{2E_0}{T}}\cos(2\pi f_i t)$,

$0 \le t \le T$, $i = 1, \ldots, M$, with M = 16 and frequency spacing $\Delta f = 1/T$ using coherent receiver. Assume the signals are transmitted through the additive white Gaussian noise channel of zero mean and power spectral density $N_0/2$. Let $E_0/N_0 = 4$ and $T = 1$.

(一)、 (4%) With coherent detection, please determine the union bound of symbol error probability.

(二)、 (4%) Following from union bound of symbol error probability, please determine the bit error probability of this 16-FSK system.

(三)、 (5%) What is the spectral efficiency (in terms of bits/Hz) of this modulation scheme? By changing the frequency spacing Δf, what is the best spectral efficiency that can be achieved?

注意：背面有試題

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五、(10%) Consider a random process: $X(t)=\sum_{k=-\infty}^{\infty} a_k p(t-kT-\Delta)$, where $\{a_k\}_{k=-\infty}^{\infty}$ is a sequence of real random variables with zero mean and $E\{a_k a_{k+m}\}=R_m,\ \forall k$. The function $p(t)$ is a deterministic real pulse-shaping function, where T is the separation between adjacent pulses; Δ is a random variable that is independent of a_k and uniformly distributed in the interval $(-T/2, T/2)$. Is $X(t)$ wide-sense stationary? Why? You need to prove your answer.

六、(15%) Consider the problem of binary signal transmission over an additive white Gaussian noise (AWGN) channel specified by $r=s+n$, where r is the received signal, $s \in \{s_0, s_1\}$ $(s_0 < s_1)$ is the transmitted signal, and n is the AWGN with zero mean and variance σ^2. Assume that the priori probabilities are:

$$\Pr\{s=s_0\}=p_0 \text{ and } \Pr\{s=s_1\}=p_1=1-p_0.$$

(一)、 (10%) Derive the optimal decision rule that minimizes the probability of error. Hint: Use the maximum a posteriori (MAP) decision criterion.

(二)、 (5%) Derive the minimum probability of error P_e.