類組：電機類　科目：通訊系統(通訊原理) (300E)　※請在答案卷內作答

Please note:

1. Try to order your answers according to problem numbers. 答案請儘量依題號順序排列。
2. Your answers may be in English or Chinese. 可用英文或中文答題。
3. If you think that the conditions given in a problem are incomplete, then make proper assumptions and state clearly your assumptions and reasons. 若你認為某題目所給的條件不完整，則請自行作適當的假設，並請敘明你的假設與理由。

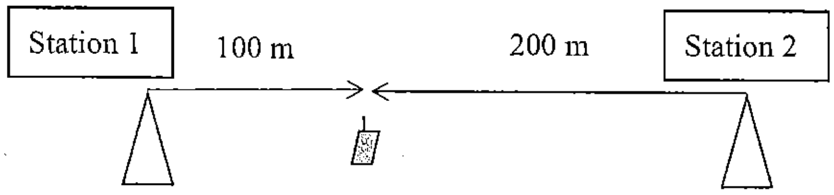
Problems:

1. (Total = 8%) Please find the frequency response and the impulse response of the linear time-invariant (LTI) system having output $y(t)$, or $y[n]$, and input $x(t)$, or $x[n]$:
   (a) (4%) A continuous-time system with $x(t)=e^{-t}u(t)$, $y(t)=e^{-2t}u(t)+e^{-3t}u(t)$, where t indicates time.
   (b) (4%) A discrete-time system with $x[n]=\left(\frac{1}{2}\right)^n u[n]$, $y[n]=\frac{1}{4}\left(\frac{1}{2}\right)^n u[n]+\left(\frac{1}{4}\right)^n u[n]$, where n is time index.

2. (Total = 5%) For each signal below, determine the largest sampling interval T_s such that aliasing does not occur.
   (a) (2%) $x_1(t)=\frac{1}{t}\sin(3\pi t)+\cos(2\pi t)$.
   (b) (3%) $x_2(t)=\frac{1}{t}\sin(3\pi t)\cos(2\pi t)$.

3. (Total = 12%) The baseband line coded signal, e.g. NRZ change, bipolar RZ, and Manchester, can be modeled by a random pulse train $x(t)=\sum_{k=-\infty}^{\infty}a_k p(t-kT-\Delta)$, where a_k is a sequence of real random variables with autocorrelation $E[a_k a_{k+m}]=R_m$, $p(t)$ is a deterministic pulse-type waveform, T is the separation between pulses, and Δ is a uniformly distributed random variable in the interval $(-T/2, T/2)$ that is independent of a_k. Suppose $a_k=g_0A_k+g_1A_{k-1}$, where g_0 and g_1 are real constants and A_k is a real random variable such that $A_k=\pm A$, where the sign is determined by a random coin toss independently from pulse to pulse for all k.
   (a) (8%) Given the pulse shape $p(t)=\Pi(\frac{t}{T})$, where $\Pi(t)$ is the "unit rectangular pulse" whose value is equal to 1 for $-0.5\le t\le 0.5$ and zero otherwise. Please determine the power spectral density of $x(t)$.
   (b) (4%) Following part (a), please plot the power spectra of the random pulse waveforms for (i) $g_0=1$ and $g_1=0$, and (ii) $g_0=-g_1=1/\sqrt{2}$. If the channel frequency response has a null at f = 0 Hz, which one is preferable? Why?

4. (Total = 15%) The following figure depicts that two base stations simultaneously transmit the same signal, at the same power and in free space, to a cellular phone which is 100 m away from the first base station and 200 m away from the second base station. The signal is a BPSK signal with the bit rate R = 1 kbps and the carrier frequency f_c = 1.5 MHz. Will the received signal be stronger in this situation than when only the first base station is transmitting to the phone? You need to give reasonable arguments to get full credits. The speed of light is 3×10^5 km/s.



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注意：背面有試題

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5. (Total = 10%) The following table summarizes the minimum Signal-to-Noise Ratio (SNR) required at the receiver to support a certain modulation scheme to maintain the Bit Error Rate (BER) below 10^{-6} on Planet X. Suppose that an engineer on Planet X has two transmission links to share the task of transmitting data; the first link results in the receiver SNR = $P_1/2$ and the second link results in the receiver SNR = $P_2/5$, with P_i being the transmitting power of the i-th link. The noises in the two links are statistically independent. Please find the minimum total transmitting power P_1+P_2 required to have the links transmitting a total of 6 bits per symbol time and maintain the BER below 10^{-6}. Note that all the SNR values given in this problem are in linear scale; that is, they are *not* in a logarithmic scale like dB.

	BPSK	4-QAM	8-QAM	16-QAM	32-QAM	64-QAM
SNR	1	1	2	4	8	16

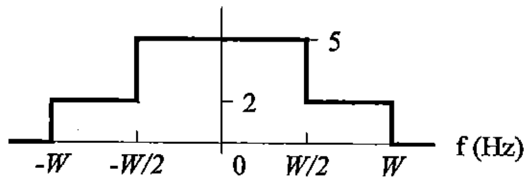
6. (Total = 8%) Consider a binary digital communication system which can be described as

$$y(t) = s(t) + n(t),$$

where $y(t)$ is the received signal at the receiver, $s(t)$ is the transmitted signal equally likely to be $s_0(t)$ or $s_1(t)$, and $n(t)$ is additive white Gaussian noise (AWGN) with zero mean and power spectral density $N_0/2$. The signal $s(t)$ and the noise $n(t)$ are assumed to be independent. The signals $s_0(t)$ and $s_1(t)$ have energy E_0 and E_1, respectively, with $E_i = \int_{-\infty}^{\infty} s_i^2(t)dt$ for i=1,2, and the correlation between $s_0(t)$ and $s_1(t)$ is given by $\rho = \int_{-\infty}^{\infty} s_0(t)s_1(t)dt$. It is known that under the same noise condition, the error probability of the optimum receiver for the binary system will be smaller if $E_0 + E_1 - 2\rho$ is larger.

Let $s_1(t) = \lambda \cdot s_0(t)$ for some unknown real number λ. Suppose now energies E_0 and E_1 must be chosen to satisfy the energy budget $E_0 + E_1 \le 2E$ (where $E > 0$), and signal $s_0(t)$ has already been designed to have energy $E_0 = \alpha \cdot E$ for a given α, where $0 \le \alpha \le 1$. Please find λ, in terms of α, that achieves the smallest possible error probability.

7. (Total = 16%) Consider a baseband digital transmission system consisting of a PAM modulator, a transmitter filter, a channel, a receiver filter, a sampling circuit, and a decision circuit. Let the channel have the frequency response as shown in the figure below (which is purely real; the imaginary part is identically zero).



(a) (4%) Let W = 3700 Hz and let the receiver filter be an ideal lowpass filter (LPF) with bandwidth $B_r > W$. There is no restriction to how the transmitter filter behaves, except that it is a linear time-invariant (LTI) system with finite output power. According to Nyquist's criterion (i.e., Nyquist's first criterion) for transmission with zero intersymbol interference (ISI), what is the maximum symbol rate (i.e., number of symbols per second) that can be transmitted over this system? Give clear reason to your answer.

(b) (4%) Following part (a), sketch the frequency response of a transmitter filter that can be used to achieve maximum-symbol-rate transmission over the channel with zero ISI. Label your plot clearly.

(c) (4%) Now consider a situation different than part (a). Here, W = 5000 Hz. In addition, the transmitter filter and the receiver filter are both ideal LPF with the same bandwidth $B > W$. What is the maximum symbol rate (in number of symbols per second) that can be transmitted over this system while satisfying Nyquist's criterion for zero ISI? You should give clear reason to your answer.

(d) (4%) Let the system of part (a) employ 8-PAM and the system of part (c) employ 16-PAM. Based on your answers to these parts, find the transmission data rate (in bits per second) of each system. Show your calculation process clearly.

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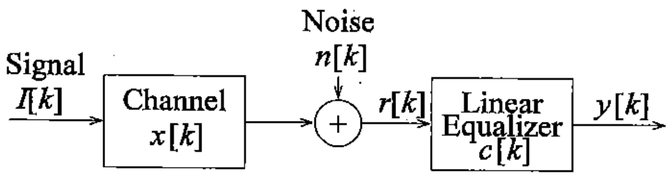
8. (Total = 9%) Consider baseband digital transmission over an imperfect channel. Assume that we can model the transmission process as a discrete-time system as shown in the figure below, where $x[k]$ and $c[k]$ are the discrete-time impulse responses of the channel and the equalizer, respectively.



Let the signal $I[k]$ be real, zero-mean, i.i.d. (independent and identically distributed), and with $E\{I^2[k]\} = 3$. Let the channel response be given by

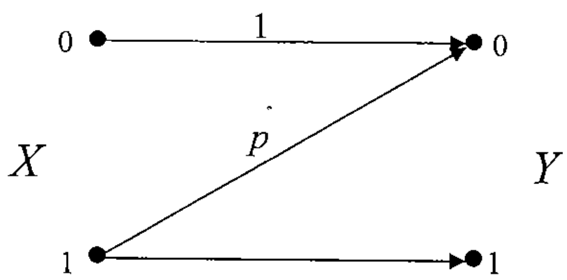
$$x[k] = \begin{cases} 1, & k = 0, \\ 0.8, & k = 1, \\ 0, & \text{otherwise.} \end{cases}$$

Let the linear equalizer contain three taps located at time indexes $-1 \le k \le 1$, that is, $c[k] = 0$ for $k < -1$ and $k > 1$. And let the equalizer be a zero-forcing (ZF) one.

(a) (5%) Suppose the desire is to result in "zero-delay operation" such that $y[k] \approx I[k]$ (where the approximation may be very inaccurate under the given conditions). It is known that the equalizer coefficients can be obtained by solving a matrix-vector equation of the form $\mathbf{Ac} = \mathbf{q}$ where $\mathbf{A}$ is a 3×3 matrix, $\mathbf{q}$ is a column vector containing three elements, and $\mathbf{c}$ is a column vector of the three unknown equalizer coefficients with $c[-1]$ on the top, $c[0]$ in the middle, and $c[1]$ at the bottom. Fill in the matrix $\mathbf{A}$ and the vector $\mathbf{q}$ with proper numerical values. Do not solve the equation.

(b) (4%) Actually, the system as described above is not causal and, hence, is not realizable. Explain why this is so. *Moreover*, please provide a simple way to resolve this issue in which the solution of the matrix-vector equation $\mathbf{Ac} = \mathbf{q}$ defined in part (a) can be re-used, without having to formulate and solve a different equation.

9. (Total = 17%) Consider the binary channel shown in the figure below, where $X \in \{0,1\}$ is the transmitted bit with $P[X = 0] = 1/3$ and $P[X = 1] = 2/3$, and $Y \in \{0,1\}$ is the observation at the output of the channel. The transition probabilities are given by $P[Y = 0 \mid X = 1] = p$, and $P[Y = 0 \mid X = 0] = 1$.



(a) (5%) Find the entropy of Y. (Express your answer in terms of p.)
(b) (5%) Does the observation of Y reduce the entropy of X? Provide your reasons in detail.
(c) (7%) Given Y, find the optimum decision rule about X that minimizes the probability of error.

(End of problems)