※選擇題請填於答案卡，非選擇題請在答案卷內作答

(一) 單選題：共 10 題，每題 4 分，答錯者每題倒扣 1 分，倒扣率即為 1/4，倒扣至本大題(即單選題) 0 分為止。

1. Consider the two electrons in a helium atom. Which of the following contradicts the Pauli exclusion principle?

(A) The two electrons must have opposite spins in the ground state of the atom.
(B) The two electrons can have parallel spins in an excited state of the atom.
(C) The two electrons can have opposite spins in an excited state of the atom.
(D) If the two electrons have parallel spins, they cannot both occupy 2p orbitals.
(E) If the two electrons have opposite spins, they can both occupy 2s orbitals.

2. Consider an intrinsic semiconductor. Which of the following is wrong?

(A) The conduction band is partially occupied at the temperature $T = 300$ K.
(B) The valence band is totally filled at $T = 0$ K.
(C) Electrons and holes can be introduced into the semiconductor by doping the semiconductor.
(D) At $T = 0$ K, electrons or holes can be introduced into the semiconductor by injecting electrical currents into the semiconductor.
(E) At $T = 0$ K, electrons and holes can be introduced into the semiconductor by exposing the semiconductor to an electromagnetic wave of any frequency.

3. Which of the following statement is correct?

(A) The wave function $\Psi(x,t) = A\sin(kx + \omega t)$ describes a particle moving in the positive x direction
(B) An electron is confined to one-dimensional box of width L and is in its ground state. A proton is confined in another one-dimensional box also of width L also in its ground state. The wave functions have the different wavelength.
(C) Consider a particle of mass m in one dimension, moving in a potential $V(x) = -k/x^{3/2}$, where $k > 0$. The ground state energy E_0 for the particle in terms of k, m and $\hbar$ is given by $\frac{27}{32}\frac{k^4 m^3}{\hbar^6}$.
(D) Consider a particle confined to the region $0 \le x \le L$ with the wavefunction $\Psi(x,t) = N\sin\left(\frac{2\pi x}{L}\right)e^{-iEt}$ and $\mathrm{E} = \frac{2\hbar^2}{mL^2}$. We need $\mathrm{N} = \frac{2}{\sqrt{L}}$ to normalize the wavefunction $\Psi(x,t)$.
(E) None of the above.

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4. In a photoelectric effect experiment on a certain metal, it is observed that incident light of wavelength 413 nm causes electrons to be ejected from the surface of metal with a maximum kinetic energy of 3.2×10^{-19} J. What is the longest wavelength of light that will eject electrons from this metal?

(A) 1.23 nm

(B) 12.3 nm

(C) 123 nm

(D) 1230 nm

(E) 1.23 mm

5. Find the drift velocity v_d of the free electron in a copper wire whose cross-sectional area is 1 mm^2 when the wire carries a current of 1 A. Assume that each copper atom contributes one electron to the electron gas. Each electron has the charge e and in the time t it travels the distance $v_d t$ along the wire. The density and molar mass of copper are 8.96 g/cm^3 and 63.5 g/mole, respectively.

(A) $v_d = 7.4\times10^{-5}$ m/s.

(B) $v_d = 4.7\times10^{-2}$ m/s.

(C) $v_d = 1.57\times10^{6}$ m/s.

(D) $v_d = 3.14\times10^{-6}$ m/s.

(E) $v_d = 9.4\times10^{-2}$ m/s.

6. Consider a gas of atomic hydrogen, which of the followings is <u>not true</u>?

(A) The number of possible states that correspond to the quantum number n is $2n^2$.

(B) The ground state energy ($n = 1$) is -13.6 eV.

(C) The relative population of the first excited state ($n = 2$) at 0°C is $4e^{-405}$.

(D) At the temperature of 10,000°C, we may expect that the number of excited atoms is about 10^{21} for a cubic meter of atomic hydrogen which contains about 2.7×10^{25} atoms at atmospheric pressure.

(E) We may expect to find $1/10$ of the atoms in the first excited state at the temperature of 3.3×10^4 K

7. Suppose you are in a spaceship travelling through the solar system at a constant speed of $v = 1.5\times10^8$ m/s relative to Mars. You fire a pulse of laser light out the back of your vessel (i.e. in the direction from where you came from). How fast does an inertial observer on Mars see the pulse leave your ship?

(A) 1.5×10^8 m/s

(B) 2.4×10^8 m/s

(C) 3×10^8 m/s

(D) 4.5×10^8 m/s

(E) Something else.

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8. Bohr model was derived as a primitive model of Hydrogen atom, which can calculate the orbital radius, wavelength, speed, period, and energies of the electron, such as r_n, λ_n, v_n, τ_n, E_n, $n = 1, 2, 3....$ Please make required modification on the original Bohr model to calculate r_1, λ_1, v_1, E_1, E_2, and v_α for a muon with $m_u = 220\ m_e$ (m_e: the rest mass of electron) captured by a proton to form a muonic atom. v_α is the photon frequency for transition from E_2 to E_1 (α-line).

(A) $r_1 = 1.256\times10^{-13}$ m, $\lambda_1 = 7.892\times10^{-13}$ m, $v_1 = 2.35\times10^{6}$ m/s, $E_1 = -3668$ eV, $E_2 = -917$ eV, $v_\alpha = 6.651\times10^{17}$ Hz

(B) $r_1 = 1.865\times10^{-13}$ m, $\lambda_1 = 1.172\times10^{-12}$ m, $v_1 = 2.35\times10^{6}$ m/s, $E_1 = -3164$ eV, $E_2 = -791$ eV, $v_\alpha = 5.737\times10^{17}$ Hz

(C) $r_1 = 2.185\times10^{-13}$ m, $\lambda_1 = 1.373\times10^{-12}$ m, $v_1 = 2.22\times10^{6}$ m/s, $E_1 = -2816$ eV, $E_2 = -704$ eV, $v_\alpha = 5.106\times10^{17}$ Hz

(D) $r_1 = 2.694\times10^{-13}$ m, $\lambda_1 = 1.693\times10^{-12}$ m, $v_1 = 2.187\times10^{6}$ m/s, $E_1 = -2672$ eV, $E_2 = -668$ eV, $v_\alpha = 4.845\times10^{17}$ Hz

(E) $r_1 = 3.068\times10^{-13}$ m, $\lambda_1 = 1.928\times10^{-12}$ m, $v_1 = 2.198\times10^{6}$ m/s, $E_1 = -2316$ eV, $E_2 = -579$ eV, $v_\alpha = 4.2\times10^{17}$ Hz

9. For an electron with a de-Broglie wavelength of 2.425×10^{-12} m, please find the correct the momentum p, kinetic energy KE, rest energy E_0, total energy E, and velocity v of the electron and phase velocity v_p.

(A) $p = 2.7324\times10^{-22}$ kg·m/s, $v = 2.9995\times10^{8}$ m/s, $KE = 4.0978\times10^{-14}$ Joule

(B) $E_0 = 8.1876\times10^{-14}$ Joule, $KE = 4.0978\times10^{-14}$ Joule, $E = 1.2285\times10^{-13}$ Joule

(C) $p = 511$ KeV/c, $KE = 212$ KeV, $v = 2.12\times10^{8}$ m/s, $v_p = 4.2388\times10^{8}$ m/s $> c$

(D) $p = 511$ KeV/c, $KE = 212$ KeV, $E = 723$ KeV, $v = 2.73\times10^{8}$ m/s, $v_p = 3.2925\times10^{8}$ m/s $> c$

(E) $p = 511$ KeV/c, $KE = 255.8$ KeV, $v = 2.9995\times10^{8}$ m/s, $v_p = 2.9965\times10^{8}$ m/s

10. You do the photoelectric effect with a big chunk of Gold (work function= 5.1 eV), which you happened to have to lying around in your room. You find that with the light source you have at hand, you get a stopping potential of 1.1 V. What is the wavelength of your light source?

(A) 150 nm
(B) 200 nm
(C) 300 nm
(D) None of the above.
(E) Cannot tell, because there is not enough information to calculate this.

注意:背面有試題

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(二) 多選題：共 10 題，每題 4 分，答錯倒扣 0.8 分，倒扣率即為 1/5，倒扣至本大題(即多選題) 0 分為止 請於答案卡上作答

11. The Michelson-Morley experiment tests if the speed of light in all inertial frames… (Please choose the following statements that are incorrect.)

(A) …is the same in air and in vacuum.
(B) …is the same in accelerating frame.
(C) …is the same in all directions.
(D) …depends on the wavelength or color.
(E) …changes when reflected by mirror.

12. The measured conductivity of copper at room temperature is $5.88\times10^{7}\ \Omega^{-1}\mathrm{m}^{-1}$ and the Fermi energy of copper is 7.03 eV. Assume that each copper atom contributes one electron to the electron gas. The density and molar mass of copper are 8.96 g/cm^3 and 63.5 g/mole, respectively. Which of the followings are correct?

(A) The electron density in copper is about 8.5×10^{28} electrons/m^3.
(B) The average time between collisions is about 2.46×10^{-14} s.
(C) The Fermi velocity is about 1.57×10^{7} m/s.
(D) The mean free path is about 3.86×10^{-7} m.
(E) All statements are correct.

13. Which of the following statements are correct?

(A) The Maxwell-Boltzmann distribution function holds for identical particles that are sufficiently far apart to be distinguishable.
(B) Bosons are particles whose spin are 0 or an integer and have wavefunctions that are asymmetric to an exchange of any pair of them.
(C) The bosons do not obey the exclusion principle and the Bose-Einstein distribution function may be exploited for them.
(D) Electrons are fermions and we shall use Fermi-Dirac statistics to study the behavior of the free electrons in a metal that are responsible for its ability to conduct electric current.
(E) None of the above.

14. Electrons are accelerated through an electric potential V and then fall on a pair of slits that have a separation of 100 nm. The resultant interference pattern indicates that the electrons have a wavelength of 1.0 nm. Which of the following statements are correct?

(A) The momentum of one of the electrons is about 6.63×10^{-25} kg$\cdot$ m/s.
(B) The accelerating electric potential V is about 1.50 Volts.
(C) The minimum spread in the momentum of electron in the direction parallel to the plane of the slits is about 5.27×10^{-28} kg$\cdot$ m/s.
(D) The velocity of electron is about 7.25×10^{4} m/s.
(E) All statements are correct.

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15. For a neutron with given kinetic energy $KE = 23.5$ MeV to calculate the momentum p, de-Broglie wave length λ_d, velocity v, and uncertainty of the position Δx, and an electron with the same Δx to calculate the KE, p, λ_d, and v of the electron.

(A) Neutron: $p = 125.5$ MeV/c, $\lambda_d = 1.482\times10^{-14}$ m, $v = 4.565\times10^{7}$ m/s, $\Delta x = 5.366\times10^{-15}$ m

(B) Neutron: $p = 105.5$ MeV/c, $\lambda_d = 1.186\times10^{-14}$ m, $v = 3.355\times10^{7}$ m/s, $\Delta x = 4.635\times10^{-16}$ m

(C) Neutron: $p = 211.5$ MeV/c, $\lambda_d = 5.863\times10^{-15}$ m, $v = 6.581\times10^{7}$ m/s, $\Delta x = 9.217\times10^{-16}$ m

(D) Electron: $KE = 26.64$ MeV, $p = 53.5$ MeV/c, $\lambda_d = 2.316\times10^{-14}$ m, $v = 1.498\times10^{8}$ m/s

(E) Electron: $KE = 106.55$ MeV, $p = 107.06$ MeV/c, $\lambda_d = 1.158\times10^{-14}$ m

16. For lattice vibration in a crystal with the atomic mass of m, provide the potential energy $U(x)$ in one dimension x to solve the wave function $\Psi(x)$, the oscillation frequency ω, the quantum energy E_n, zero-point energy ε_0, and average energy $\langle E\rangle$ at temperature T under the condition of the probability function $P(E_n) = e^{-E_n/k_eT}$ to calculate $\langle E\rangle$ at $T \cong 0$ and $\langle E\rangle$ at very high temperature.

(A) $\Psi(x) = (2^n n!)^{\frac{1}{2}} H_n(x) e^{-(m\omega x)^2/2\hbar}$, $\omega = \sqrt{k/m}$, $H_0(x) = 1$, $H_1(x) = 2x$

(B) $\Psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} (2^n n!)^{-\frac{1}{2}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right) e^{-m\omega x^2/2\hbar}$, $\omega = \sqrt{k/m}$

(C) $\Psi(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}} (2^n n!)^{-\frac{1}{2}} H_n(x) e^{-m\omega x^2/2\hbar}$, $\omega = \sqrt{k/m}$, $H_0(x) = 1$, $H_1(x) = 2x$

(D) $U(x) = \frac{1}{2}kx^2$, $\langle E\rangle \cong \frac{1}{2}\hbar\omega$ at $T \cong 0$, $\langle E\rangle \cong k_BT$ at very high T

(E) $U(x) = \frac{1}{2}kx^2$, $\langle E\rangle = \frac{1}{2}\hbar\omega + \frac{\hbar\omega}{e^{\hbar\omega/k_BT}+1}$

17. The wavelength of a beam of light is increased but the light's intensity remains unchanged. Which of the followings are true?

(A) There are fewer photons per second.
(B) There are more photons per second.
(C) Each photon has less energy.
(D) Each photon has more energy.
(E) Nothing happens to the photon number because the light is a wave.

18. Consider a p-n diode. Which of the following are correct?

(A) Near the p-n junction, electrons and holes diffuse and recombine, resulting in a deletion region.
(B) When forward based, there will be electrons flowing from n side to p side.
(C) When forward biased, there will be holes flowing from p side to n side.
(D) When unbiased, the Fermi energy is aligned throughout the diode.
(E) When unbiased, the conduction band edge is aligned throughout the diode.

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19. Consider the angular momentum of an electron in the 2Pz orbital of hydrogen atom. Which of the following are wrong?

(A) The electron has a nonvanishing projection of angular momentum in the z direction.

(B) If the projection of electron angular momentum in the x direction is measured, it always yields zero.

(C) If the total electron angular momentum is measured, it yields a distribution with a finite width.

(D) If the projection of electron angular momentum in the z direction is measured, it always yields an identical value.

(E) If a magnetic field in the z direction is applied, it will shift the electron energy.

20. For the following normalized real wavefunctions, $\Psi_A(x)$, $\Psi_B(x)$ and $\Psi_C(x)$, versus x for a particle in which of the following statements are correct?

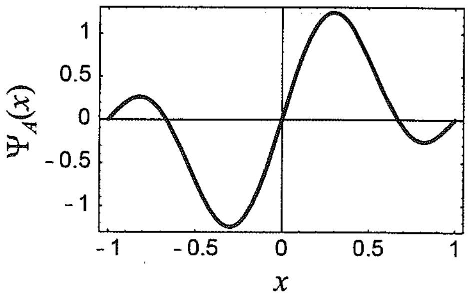
(A) The wavefunction $\Psi_B(x)$ will yield the largest value for $\langle x \rangle$.

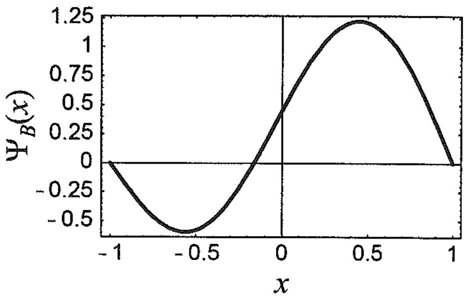
(B) The wavefunction $\Psi_C(x)$ will yield the largest value for $\langle x^2 \rangle$.

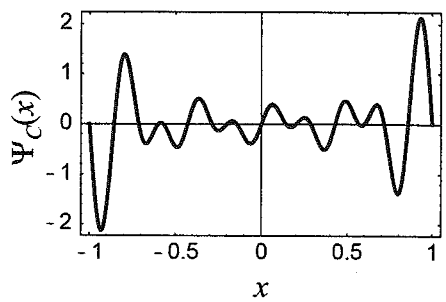
(C) The wavefunction $\Psi_C(x)$ will correspond to the highest kinetic energy.

(D) All wavefunctions $\Psi_A(x)$, $\Psi_B(x)$ and $\Psi_C(x)$ will yield the largest value for $\langle x \rangle$.

(E) All wavefunctions $\Psi_A(x)$, $\Psi_B(x)$ and $\Psi_C(x)$ will yield the largest value for $\langle x^2 \rangle$.







注意:背面有試題

(三) 非選擇題：共 2 題，每題 10 分

21. (10%) Prove that the electron magnetic moment is $\mu = -\left(\frac{e}{2m}\right)L$, where L is angular momentum.

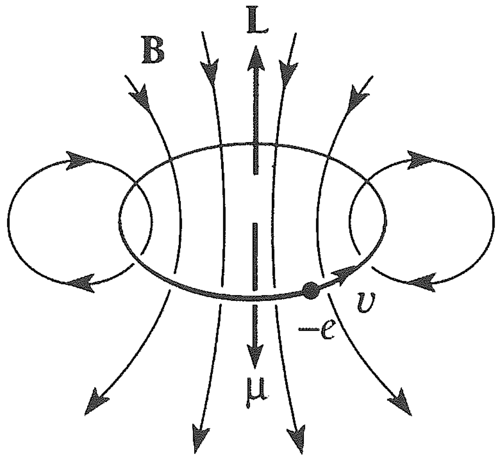


Figure. Magnetic moment of an orbiting electron of angular momentum L.

22. (10%) Specific heat of solid C_V.

(a) (5%) Determine the Einstein's formula of specific heat of solid C_V for a solid with Avogadro's number N_0 of atoms.

(b) (5%) Determine the Einstein's formula C_V at high temperature.

Electron rest mass: $m_e = 9.1095\times10^{-31}$ kg

Proton rest mass: $m_p = 1.6726\times10^{-27}$ kg

Neutron rest mass: $m_n = 1.6750\times10^{-27}$ kg

Hydrogen atomic mass: $M_H = 1.6736\times10^{-27}$ kg

Light velocity: $c = 2.998\times10^{8}$ m/s

Electron charge: $e = 1.602\times10^{-19}$ Coul

Planck's constant: $h = 6.626\times10^{-34}$ J·s $= 4.1361\times10^{-15}$ eV·s

Permittivity of free space: $\varepsilon_0 = 8.854\times10^{-12}$ F/m

Boltzmann's constant: $k = 1.381\times10^{-23}$ $J/K = 8.617\times10^{-5}$ eV/K