類組：電機類　科目：訊號與系統(300B)　　共 3 頁 第 1 頁

計算題（計算題應詳列計算過程，無計算過程者不予計分）

一、(25%)

A system has the following input and output relationship:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

(一) (6%) Show that this is an LTI system.

(二) (4%) **Obtain** the impulse response $h[n]$ for this system. **Plot** $h[n]$.

(三) (3%) Can it be implemented in practice? If your answer is yes, explain how and **plot** the block diagram. If your answer is no, explain your answer.

(四) (4%) Is the system invertible? If your answer is yes, **obtain** the inverse system's impulse response and **plot** it. If not, explain your answer.

(五) (1%) Is the system memoryless?

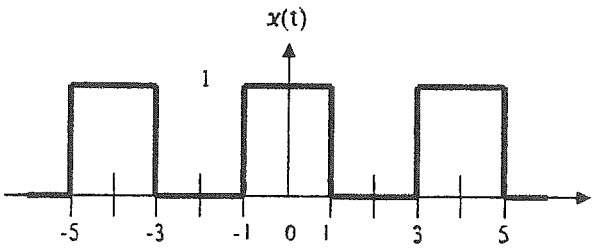
(六) (1%) Is the system causal?

(七) (6%) Now the input and output relationship is changed as follows ($\rho < 1$). Obtain the output $y[n]$ if the input signal is $e^{j\frac{2\pi}{8}n}$. Express your answer using polar coordination.

$$y[n] = \sum_{k=0}^{\infty} \rho^k x[n-k]$$

二、(15%)

Suppose that the input $x(t)$ to an ideal frequency-selective filter is the square wave depicted below.



The Fourier series representation of this waveform is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad -\infty < t < \infty$$

(一) (7%) Determine the Fourier coefficients a_k for this signal.

(二) (8%) State whether or not it is possible that the corresponding output of the LTI system could be the signal

$$y(t) = 2\cos\left(\frac{2\pi t}{4}\right) \quad -\infty < t < \infty$$

If it is impossible, state why. If it is possible, **determine** the frequency response $H(j\omega)$ of an ideal frequency-selective filter (lowpass, highpass, or bandpass) such that the output is as specified. **Draw** a carefully labeled sketch of $H(j\omega)$ showing the **gain** and **cutoff frequencies** for the filter.

三、(15%)

A causal LTI system is described by the following difference equation

$$y[n] - ay[n-1] = bx[n] + x[n-1]$$

where a is real and less than 1 in magnitude.

(一) (5%) Find a value of b such that the frequency response of the system $H(e^{j\omega})$ represents an *all-pass system*, which does not attenuate the input $e^{j\omega n}$ for any value of ω.

(二) (10%) Based on the above result, roughly **sketch** $\angle H(e^{j\omega}), 0 \le \omega \le \pi$, when $a = -\frac{1}{2}$. **Find and plot** the output of this system when the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

where $u[n]$ is the unit-step function.

四、(5%)

Find the even and odd components of each of the following signals.

(一) (2%) $x(t) = 1 + t + 2t^2 + 7t^3 + 5t^4$

(二) (3%) $x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$

五、(15%)

Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in the following figure. Perform the following calculations **without** evaluating $X(e^{j\omega})$.

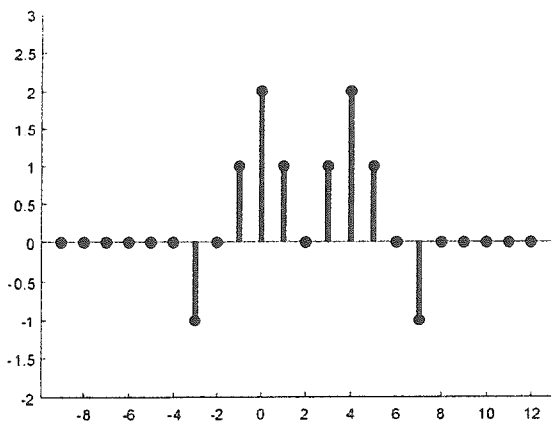
(一) (3%) Evaluate $X(e^{j0})$

(二) (3%) $X(e^{j\omega}) = |X(e^{j\omega})|e^{j\theta(e^{j\omega})}$, find $\theta(e^{j\omega})$

(三) (3%) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$

(四) (3%) Evaluate $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2\,d\omega$

(五) (3%) Evaluate $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$



注意：背面有試題

六、(10%)

Assume the system is **causal** and its Laplace transform is

$$H(s) = \frac{3s+4}{(s+1)(s+2)^2}$$

Its inverse Laplace transform can be expressed as

$$h(t) = Ae^{-t}u(t) + Be^{-2t}u(t) + Cte^{-2t}u(t).$$

Which statements are **true (multiple choices)**? No need to provide detailed process. Negative points for wrong choices until zero.

(一) $A = 1$

(二) $B = 1$

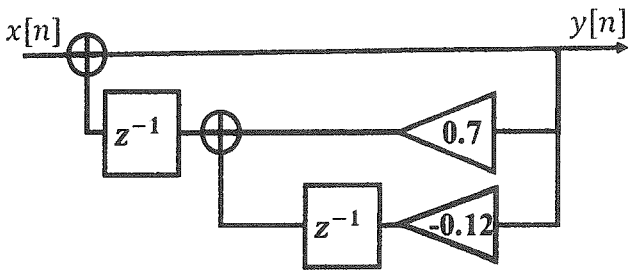
(三) $C = 2$

(四) The frequency response of $H(j1)$ (*i.e.*, $\omega = 1$) has the magnitude around 0 ± 3 dB.

(五) The frequency response of $H(j20)$ (*i.e.*, $\omega = 20$) has the magnitude around -20 ± 3 dB.

七、(15%)

Consider the below LTI system



(一) (6%) Find the transfer function (*i.e.*, $H(z)$) and ROC if it is a causal system.

(二) (5%) Find the impulse response (*i.e.*, $h[n]$) if it is a causal system.

(三) (4%) It can be a causal or noncausal system. Discuss the system stability.