類組：電機類　科目：訊號與系統(300B)　　共 3 頁 第 1 頁

※請在答案卷內作答

一、(5%) Consider a discrete-time system where x[n] is the input, y[n] is the output and the input-output relationship can be expressed as follows.

y[n] = (x[n-3]+x[n-2]+x[n-1]+x[n]+x[n+1]+x[n+2]+x[n+3])/7

Can the system be potentially implemented in real time? justify your answer. Note that "real time" here means that given an input $x[n_0]$ at time n_0, the system can provide the output $y[n_0]$ at time n_0 immediately if the computational latency can be neglected.

二、(10%) For each of the following input-output relationships, please determine whether the corresponding system is linear, time invariant or both with a detailed explanation.

(一) (2%) $y[n] = x^2[n-2]$

(二) (2%) $y(t) = t^2 x(2-t)$

(三) (2%) $y[n] = x[2n]$

(四) (2%) $y[n] = x[2-n]$

(五) (2%) $y[n] = \sum_{k=-\infty}^{2n} x[k]$

三、(15%) Please determine the time-domain signal x [n] from the Discrete time Fourier Series (DTFS) coefficients,

$X[\mathrm{k}] = \sum_{\mathrm{m}=-\infty}^{\infty}(-\mathrm{j})^{\mathrm{m}}(\delta[k-2\mathrm{m}] + \delta[k+3\mathrm{m}])$.

Hint: DTFS pair $x[n] \xleftrightarrow{DTFS;\ \Omega_0=\frac{2\pi}{N}} X[k]$

$$x[n] = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 n} = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n},\ where\ X[k] = \sum_{\ell=-\infty}^{\infty} a_{k+\ell N}$$

$$X[k] = \frac{1}{N}\sum_{k=0}^{N-1} x[n] e^{-jk\Omega_0 n}$$

四、(10%) Let $x[n]$ be a periodic sequence with period N and Fourier coefficients a_k. Express the Fourier coefficients for each of the following signals in terms of a_k:

(一) (5%) $x[n] + x\left[n + \frac{N}{2}\right]$ (Assume that N is even.)

(二) (5%) $(-1)^n x[n]$ (Assume that N is odd.)

五、 (10%) Find the impulse response of a system with the following frequency response:

$$\frac{\sin(\omega)\sin^2(3\omega)}{\omega^2}.$$

六、 (10%) Determine the Fourier coefficients of the following continuous-time signal:

$$\frac{1}{5-4\cos(2\pi t)}$$

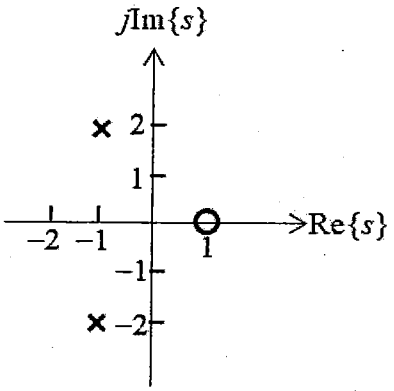
七、(15%) Given the pole/zero plot of a causal system below, determine

(一)(4%) The system function $H(s)$.

(二)(4%) The impulse response.

(三)(4%) The output signal for the input $x(t)=\cos(t)$.

(四)(3%) The differential equation relating the input $x(t)$ to the output $y(t)$.



八、(10%)

(一) (5%) Prove the convolution property of the z transform, $Z\{x[n]*h[n]\} = X(z)H(z)$, and specify the change of region of convergence.

(二) (5%) Consider a discrete-time system $H(z) = 1/(1+az^{-1})$. Compare the difference in frequency response between the cases of $a = 0.9$ and $a = -0.9$ by sketching the magnitude and phase response.

九、 (15%) The discrete-time sequence x[n] is bandlimited to $2\pi/3$ radians in angular frequency and you can find its discrete time Fourier transform (DTFT) spectrum in the frequency range $[-\pi, \pi]$ in the left panel of figure (a). We would like to use the fractional sampling rate converter shown in figure (b) to change the bandwidth of x[n] to π radians (as illustrated in the right panel of figure (a)).

(一) (5%)Determine the smallest values of I and D in figure (b) to achieve the goal mentioned above.

(二) (10%) For the fractional sampling rate converter in figure (b), can we exchange the interpolator and decimator in this case while still achieving the same goal mentioned above? Justify your answer by spectral illustration, that is, by drawing and comparing the resultant DTFT spectra.