類組：電機類　科目：離散數學(300C)　※請在答案卷內作答

一. 填空題(共20題，每題4分：合計80分)

答題說明：
1. 請依題號順序書寫於答案卷，並清楚標註題號。
2. 題號1-10題目詢問內容描述正確與否(題目前標註(T or F)者)，認為描述正確者書寫T，錯誤者書寫F。其餘答案一律不給分。
3. 其餘題目(11-20)請直接書寫答案，無需計算過程。

1. (**T** or **F**) Given a set $\{P, Q, R\}$ of Boolean variables and the following well formed formulae (wff)
$(P \vee Q \vee R) \wedge (\overline{P} \vee \overline{Q} \vee \overline{R}) \wedge (\overline{P} \vee Q) \wedge (\overline{Q} \vee R)$
is *satisfiable*.

2. (**T** or **F**) The halting problem (HPS) on input i is defined by

$$HPS_i = \begin{cases} 1, & \text{if there is at least one } x \in N \text{ for which } P_i(x) \text{ halts;} \\ 0, & \text{otherwise.} \end{cases}$$

Then, HPS_i is a *computable* function.

3. (**T** or **F**) $2^{\{0,1\}^*}$ is *countably infinite*.

4. (**T** or **F**) For a set, the $\cap$ operation is neither left nor right distributive with respect to the $\times$ operation.

5. (**T** or **F**) The complete bipartite graph $K_{3,3}$ is planar

參考用

注意:背面有試題

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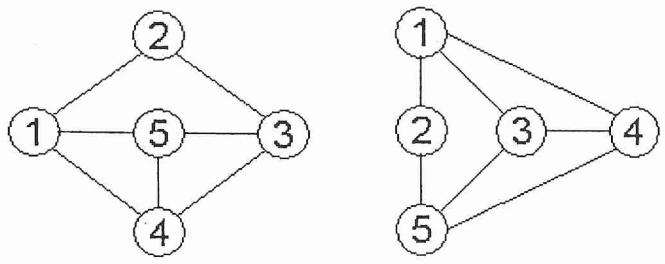
6. (**T** or **F**) The following two graphs are isomorphic.



7. (**T** or **F**) If a graph G is an undirected graph, the number of vertices with odd degree is even.

8. (**T** or **F**) An additive group can be isomorphic to a multiplicative group.

9. (**T** or **F**) Let Ackermann's function A be defined as

$$A(x,y) = \begin{cases} y+1, & \text{if } x = 0 \\ A(x-1,1), & \text{if } y = 0 \\ A(x-1, A(x,y-1)), & \text{otherwise} \end{cases} , \forall x, y \geq 0, x, y \in \mathbb{Z}$$

Ackermann's function A is a total function from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$.

10. (**T** or **F**) If $R1$ and $R2$ are two equivalence relations, then $R1 \cup R2$ is also an equivalence relation.

參考用

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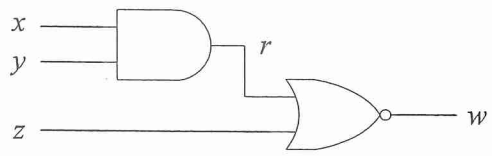
類組：電機類　科目：離散數學(300C)　※請在答案卷內作答

11. Please derive the **conjunctive normal form (CNF)** for the following circuit:



Note that the negated of a Boolean variable x should be represented as $\bar{x}$.

12. Solve the following recurrence:
$nf(n) = (n-1)f(n-1) + (n-2)f(n-2) + 2$,
where $\forall n \leq 2$ and $f(0) = f(1) = 0$
(Hint: use the characteristic roots method)

13. Derive a formula for $\sum_{i=1}^{n} i^3 \binom{n}{i}$ where $n \in \mathbb{N}$.

14. Given a partially ordered set $P = \{2, 3, 5, 7, 11, 13, 17\}$ with the partial order $\preceq$ as "m divides n", please draw the corresponding Hasse diagram.

15. Let $A = B = C = \mathbb{R}$ where $\mathbb{R}$ is the set of all real numbers. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined respectively by
$$f(x) = 2x^2 + 3$$
$$g(x) = 5x - 1,$$
please derive $f \circ g \circ f(x) : \mathbb{R} \to \mathbb{R}$.

參考用

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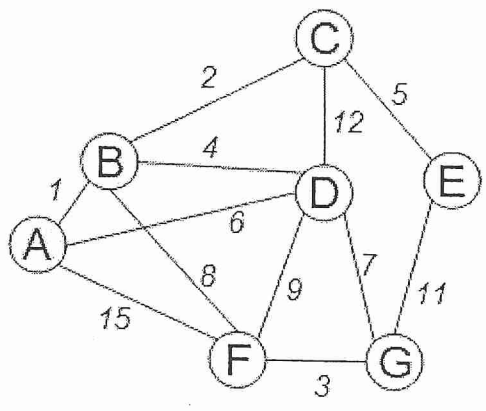
類組：電機類　科目：離散數學(300C)　※請在答案卷內作答

16. How many solutions are there , in positive integers, to the following equation:

$$x_1 + x_2 + \cdots + x_{10} = 15$$

17. Derive the cost of the minimum spanning tree for the following graph



18. Solve the following recurrence relation and give a Θ-bound for

$$T(n) = \sqrt{n}T(\sqrt{n}) + n$$

19. Solve the following recurrence relation and give a Θ-bound for

$$T(n) = 49T(\tfrac{n}{25}) + n^{\frac{3}{2}}\lg n \text{ where } \lg n \equiv \log_2 n$$

20. Solve the following recurrence relation and give a Θ-bound for

$$T(n) = \begin{cases} 8T(\frac{n}{2}) - n^3 + 7n^4 + 5, & n > 1 \\ 1, & n = 1 \end{cases}$$

參考用

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類組：電機類　科目：離散數學(300C)　※請在答案卷內作答

二. 問答/計算題(共3大題：合計20分)

答題說明：1. 請依題號順序書寫於答案卷，並清楚標註題號。
2. 每題題目前說明小題配分。例如：(5分+5分)即代表本題中第一小題五分與第二小題五分。以此類推。

21. (5分)

Let $\oplus$ denote the exclusive operator $x \oplus y = (x \vee y) \wedge (\bar{x} \vee \bar{y})$. Show that

$$\overline{\oplus_{1 \le i \le k} X_i} = \overline{X_j} \oplus X_1 \oplus \cdots \oplus X_{j-1} \oplus X_{j+1} \oplus \cdots \oplus X_k.$$

22. (5分)

Please show the following two equations,

$$a_1 + a_2 + \cdots + a_5 = 11 \text{ and } b_1 + b_2 + \cdots + b_{12} = 4,$$

have the same number of solutions in nonnegative integers

23. (5分+5分)

Given a character set and the frequency of each character,

character	*frequency*
A	24
E	16
I	35
O	18
U	7

(1) Find a Huffman code for the character set using the given frequencies. Show your derivation.

(2) Decode the message 01000110001011