所別：物理學系暨生物物理研究所碩士班　科目：應用數學

1. (10%) For a particle moving in a circular orbit $\vec{r} = \hat{i}rCos(\omega \cdot t) + \hat{j}rSin(\omega \cdot t)$.
Evaluate (a) $\vec{r} \times \frac{\partial \vec{r}}{\partial t}$ and (b) $\frac{\partial^2 \vec{r}}{\partial t^2} + \omega^2 \vec{r}$.

2. (10%) A point in space may be specified as (ρ_1, φ_1, z_1) in circular cylindrical coordinates system. Sketch the unit vectors $\vec{\rho}_0$, $\vec{\varphi}_0$, and $\vec{z}_0$ at that point, and obtain the transformation relations between (ρ_1, φ_1, z_1) and (x_1, y_1, z_1).

3. (10%) If matrices A and B are diagonal, show that A and B commute.

4. (10%) Obtain the Fourier series of $f(x) = x^2$.

5. (15%) By application of Green's theorem in three dimension, namely $\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \oint_S \left(\phi \frac{\partial \psi}{\partial n} - \psi \frac{\partial \phi}{\partial n} \right) dS$, show that $\nabla^2 \left(\frac{1}{r} \right) = -4\pi\delta(r)$.

6. (15%) The quadric surface $x^2 + 6xy - 2y^2 - 2yz + z^2 = 24$ may be represented in matrix form as $(x \quad y \quad z) \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 24$. Find (a) the equation of the quadric surface relative to its principal axes (x', y', z'), and (b) the rotation matrix that rotates axes (x, y, z) into (x', y', z').

7. (15%) By considering a function $\phi(x)$, which in the interval $x = [-L, L]$ is represented by the Fourier sum $\phi(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n Cos \frac{n\pi x}{L} + b_n Sin \frac{n\pi x}{L} \right)$. Show that the quantity $f(x) = \frac{1}{2L} + \frac{1}{L} \sum_{n=1}^{\infty} Cos \frac{n\pi x}{L}$ incorporates properties of the delta function.

8. (15%) Using complex analysis to show that $I = \int_0^{2\pi} \frac{d\theta}{1 - 2\alpha Cos\theta + \alpha^2} = \frac{2\pi}{1 - \alpha^2}$, where $|\alpha| < 1$, by setting $Z = e^{i\theta}$.