所別：物理學系碩士班 不分組 科目：近代物理

1. Explain the followings (4 pts each)
   (a) Compton effect
   (b) Wave-particle duality
   (c) Uncertainty principle
   (d) Bohr's model of the hydrogen atom
   (e) Shroedinger equation and wave function
   (f) Spin-orbit interaction
   (g) The Stern-Gerlach experiment

2. (a) (3 pts) Draw a graph showing spectrum of black body radiation of two temperatures T_1 and T_2, with $T_2 > T_1$.
   (b) (4 pts) Consider a cubic box with size $L \times L \times L$ filled with electromagnetic radiations. Show that the number of allowed values in frequency range ν and $\nu + d\nu$ is

$$N(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu.$$

   Here $V = L^3$ and c is the speed of light.
   (c) (3 pts) In classical picture, the probability $P(\epsilon)$ for each frequency carrying energy ϵ is proportional to $\exp(-\epsilon/kT)$. Show that the average energy for each frequency is kT, and derive the Rayleigh-Jeans formula for blackbody radiation.
   (d) (5 pts) What is the Planck's postulate about the energy ϵ?
   (e) (7 pts) Derive the Planck's formula for blackbody radiation.

3. (a) (3 pts) Write down the one dimensional time-dependent Schroedinger's equation for the wave function $\Psi(x,t)$, with a potential $V(x)$.
   (b) (4 pts) Use the separation of variable

$$\Psi(x,t) = \psi(x)\phi(t)$$

   to derive the time-independent Schroedinger's equation for $\psi(x)$,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

   What is $\phi(t)$?

注意：背面有試題

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(c) (10 pts) Consider a square-well potential of

$$V(x) = \begin{cases} V_0 & x < -a/2 \\ 0 & -a/2 < x < a/2 \\ V_0 & x > a/2 \end{cases}$$

Calculation the wave functions $\psi(x)$ and eigenvalues E for $E < V_0$.

(d) (3 pts) Make a graph showing some of the calculated wave functions.

(e) (5 pts) Consider another potential of

$$V(x) = \begin{cases} 0 & x < -a/2 \\ V_0 & -a/2 < x < a/2 \\ 0 & x > a/2 \end{cases}$$

When a particle approaches the potential from $x = -\infty$ with an energy E which is smaller than V_0, explain why there is a tunneling probability P_t for the particle to tunnel through the potential barrier.

(f) (5 pts) Estimate the functional dependence of P_t on E, V_0, and a.

4. For a system with two states $|1\rangle$ and $|2\rangle$, we can write its state vector as

$$|\psi\rangle(t) = C_1(t)|1\rangle + C_2(t)|2\rangle .$$

(a) (5 pts) Since $|\psi\rangle$ follows

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle ,$$

show that C_1 and C_2 follow the equations

$$\begin{aligned} i\hbar \frac{dC_1}{dt} &= H_{11}C_1 + H_{12}C_2 \\ i\hbar \frac{dC_2}{dt} &= H_{12}C_1 + H_{22}C_2 \end{aligned}$$

Here $H_{ij} \equiv \langle i| H |j\rangle$.

(b) (5 pts) Assume that $H_{11} = H_{22} = E_0$ and $H_{12} = H_{21} = -A$, find the solutions of $C_1(t)$ and $C_2(t)$.

(c) (5 pts) Find the stationary states $|I\rangle$ and $|II\rangle$. (A stationary state means a state with a definite energy, i.e., if $|\psi\rangle(t=0) = |I\rangle$, then $|\psi\rangle(t) = \exp(-iE_I t/\hbar)|I\rangle$.) What are E_I and E_{II}?

(d) (5 pts) Now if $H_{11} = E_0 + \epsilon$ and $H_{22} = E_0 - \epsilon$, what are E_I and E_{II}? Make a graph showing E_I and E_{II} as functions of ϵ.