國立中央大學九十一學年度碩士班研究生入學試題卷

所別：物理學系 不分組 科目：古典物理 共 1 頁 第 1 頁

You must show the steps clearly in order to earn credits.

1. Find the center-of-mass of a quadrant of a thin elliptical section (四分之一薄橢圓) of uniform mass density σ. Taking the long and short axes of the ellipse to be a and b, respectively. **(10%)**

2. Consider a pendulum consisting of a mass m suspended by a weightless string of length l in a resistless space.
   (a) Obtain the Lagrangian of the system. **(5%)**
   (b) Obtain the Lagrangian's equation of motion. **(5%)**
   (c) The string was set at a small angle θ_0 apart from the vertical line. Determine its motion after setting free. **(10%)**

3. A problem on electrostatic potential.
   (a) Obtain the Poisson's equation in free space for electrostatic potential Φ due to a charge distribution $\rho(\vec{r})$. **(5%)**
   (b) Determine the Φ inside and outside a uniformly charged infinite cylinder of radius a_0 and volume charge density α, oy solving the Poisson's equation and taking the zero potential to be at the center of the cylinder. **(10%)**

4. Consider a simple media of permittivity α, permeability β, and conductivity σ.
   (a) Describe the origin of displacement current. **(5%)**
   (b) Write down the Maxwell's equations in the media. **(5%)**
   (c) Obtain the vector wave equation for the $\vec{E}$ field in regions where no free charges are present. **(5%)**
   (d) Based on the equations obtained in (c), describe how a plane EM wave will propagate in (i) a non-conducting and (ii) a conducting media. Sketch the waveform. **(10%)**

5. General concept of thermodynamics
   (a) Define isothermal process. **(2%)**
   (b) Define adiabatic process. **(2%)**
   (c) Give the zeroth law of thermodynamics. **(2%)**
   (d) Give the first law of thermodynamics. **(2%)**
   (e) Give the second law of thermodynamics. **(2%)**

6. Consider N identical, non-interacting harmonic oscillators of angular frequency ω.
   (a) Write down the allowed energy level scheme and then obtain the number of microstates accessible for the system of energy E. **(10%)**
   (b) Show that the temperature of the system can be expressed as

$$\frac{1}{T}=\frac{k_B}{\hbar\omega}\ln\left\{\frac{E+\frac{1}{2}N\hbar\omega}{E-\frac{1}{2}N\hbar\omega}\right\},$$ where k_B is the Boltzmann constant. **(10%)**

參考用

Some useful vector identities:

$$\nabla\times\left(\nabla\times\vec{A}\right)=\nabla\left(\nabla\cdot\vec{A}\right)-\nabla^2\vec{A}$$

$$\nabla\cdot\left(\nabla\times\vec{A}\right)=0$$

$$\nabla\times\left(\nabla V\right)=0$$