

# 國立中央大學九十學年度碩士班研究生入學試題卷

所別: 物理學系 不分組 科目: 近代物理 共 2 頁 第 1 頁

1. (a) Explain the Bohr's two assumptions and use these assumption to derive the total energy of an electron:

$$E_n = -\frac{mZ^2e^4}{(4\pi\epsilon_0)^2 2\hbar^2 n^2} \quad n = 1, 2, 3, \dots (15\%)$$

(b) What is the Planck's postulate and use the formula  $\langle \epsilon \rangle = \frac{\sum_0^{\infty} \epsilon P(\epsilon)}{\sum_0^{\infty} P(\epsilon)}$  to derive Planck's expression for the average

energy  $\langle \epsilon \rangle$  and also his blackbody spectrum. Where  $P(\epsilon) = \frac{e^{-\epsilon/kT}}{kT}$ . (15%)

2. Determine Planck's constant  $h$  from the fact that the minimum x-ray wavelength produced by 40 keV electrons is  $3.11 \times 10^{-11} \text{m}$ . ( $e: 1.6 \times 10^{-19} \text{coulomb}$ ,  $c: 3 \times 10^8 \text{m/s}$ ) (10%)

3. Find the possible values of the total spin  $s$ , the total orbital angular momentum  $\ell$ , and the total (spin + orbital) angular momentum  $j$  for a configuration with three optically active electrons of quantum numbers  $\ell_1 = 1, \ell_2 = 2$ , and  $\ell_3 = 4$ . (10%)

4. (a) The infinite square well and the two lowest states, one wave function with the lowest state is sketched as the figure 1. Please sketch the second wave function. A finite square well with two energy states is shown in figure 2 and sketch two corresponding graphs of the two wave functions. (10%) (b) A potential well with a step is shown in figure 3 and draws the wave function for fifth energy level in this "step well". (10%)

5. (a) Verify that the wave function

$$A \sin \frac{2\pi x}{a} e^{-Et/\hbar} \quad -a/2 < x < +a/2$$

$$\Psi(x, t) =$$

$$0$$

$$x < -a/2 \quad \text{or} \quad x > +a/2$$

- is a solution to the Schroedinger equation in the region  $-a/2 < x < +a/2$  for a particle which moves freely through the region but which is strictly confined to it. (6%) (b) Also determine the value of the total energy  $E$  of the particle in this first excited state of the system. (6%) (c) Normalize the wave function, by adjusting the value of the multiplicative constant  $A$  so that the total probability of finding the associated particle somewhere in the region of length  $a$  equals one. (6%) (d) Calculate the expectation value of  $x$  and  $x^2$  for the particle associated with the wave function. (6%) (e) Calculate the expectation value of  $p$  and  $p^2$  for the particle associated with the wave function. (6%)

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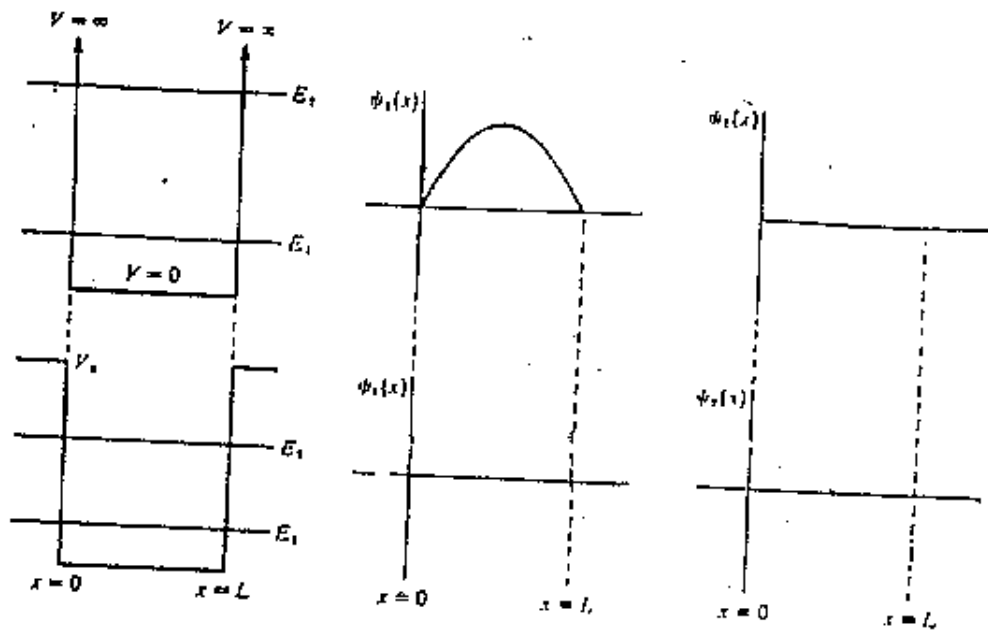


Fig. 1. (Top)      Fig. 2. (Bottom)

