國立中央大學八十五學年度碩士班研究生入學試題卷

所別：物理研究所　不分組　　科目：應用數學　　共 1 頁 第 1 頁

1. [10%] Prove Jacobi's identity for vectors $\vec{A}, \vec{B}$, and $\vec{C}$:

$$\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B}) = 0.$$

2. [10%] Evaluate the following integral around the circle $x^2+y^2=1$, counter-clockwisely,

$$\oint (x^3-3yx^2+y^3)dx+(y^2-2x^3)dy \ .$$

3. [20%] Find the general solutions to the following differential equations,

(a) $x\frac{dy}{dx}+(1+x)y=e^x$.

(b) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^x$.

4. [10%] It is well known that the Hermitian matrices have real eigenvalues and the eigenvectors belonging to different eigenvalues are orthogonal to each other. What are the characteristics of the eigenvalues and eigenvectors of *unitary* matrices ? Prove your answere.

5. [15%] Solve the one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$, with the boundary condition $u(0,t)=u(L,t)=0$.

6. [10%] Find the complex Fourier series for a periodic function,

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi. \end{cases}$$

7. [15%] Using the residue method to evaluate the improper integral,

$$\int_0^\infty \frac{1+x^2}{1+x^4}dx$$

8. [10%] Find the transformation $w=f(z)$ which maps the angular region : $-\pi/4 \le arg(z) \le \pi/4$ onto the unit disk $|w| \le 1$, in such a way that the three points $z_1=0, z_2=1, z_3=2$ are mapped to $w_1=-1, w_2=0, w_3=1$ respectively.