

所別：天文研究所碩士班 不分組 科目：應用數學

1. (10%)

$y$  is a function of  $x$  and satisfied:

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0, \quad (1)$$

$y(0) = 2$  and  $dy/dx(0) = 3$ .

Please calculate the solution  $y(x)$ .

2. (15%)

$y$  is a function of  $x$  and satisfied:

$$\frac{d^2y}{dx^2} + \beta y = 0, \quad (2)$$

$y(0) = y(\pi/2) = 0$  and  $0 \leq x \leq \pi/2$ .  $\beta$  can be any real number. Please calculate the possible solution  $y(x)$ .

3. (20%)

$U$  is a function of  $x$  and  $t$  and satisfied:

$$\frac{\partial U}{\partial x} = 2 \frac{\partial U}{\partial t} + U, \quad (3)$$

$U(x, 0) = 6e^{-3x}$ ,  $x > 0$ ,  $t > 0$ . Please calculate the possible solution  $U(x, t)$ .

(You can use Laplace transform and inverse Laplace transform.)

注意：背面有試題

參考用

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4. (10%)

Assume there are three points in three dimensional space:  $P, Q, R$  and the coordinates of  $P$  is  $(2, 3, 5)$ , the coordinates of  $Q$  is  $(4, 3, -1)$ , the coordinates of  $R$  is  $(3, 6, 4)$ . Please calculate the area of the triangle  $\triangle PQR$ .

5. (10%)

$$\mathbf{F} = (x + 3y) \mathbf{i} + (y - 2z) \mathbf{j} + (x + \alpha z) \mathbf{k}$$

Calculate the value of  $\alpha$  to vanish the Divergent of the vector  $\mathbf{F}$ , that is  $\nabla \cdot \mathbf{F} = 0$

6. (35%)

$$(A) \mathbf{G} = 3xy\mathbf{i} + yz\mathbf{j} + x^2y\mathbf{k}$$

(A.1) Please show that whether there is a function  $\phi(x, y, z)$  such that the gradient of  $\phi(x, y, z)$  equals to  $\mathbf{G}$ , that is  $\nabla\phi(x, y, z) = \mathbf{G}$ , or not.

(A.2) If this function  $\phi(x, y, z)$  exists, please solve and write down this function.

$$(B) \mathbf{F} = (2xyz^3)\mathbf{i} + (x^2z^3 + \cos z)\mathbf{j} + (3x^2yz^2 - y \sin z)\mathbf{k}$$

(B.1) Please show that whether there is a function  $\psi(x, y, z)$  such that the gradient of  $\psi(x, y, z)$  equals to  $\mathbf{F}$ , that is  $\nabla\psi(x, y, z) = \mathbf{F}$ , or not.

(B.2) If this function  $\psi(x, y, z)$  exists, please solve and write down this function.

