從以下八題中任意選擇五題，每題廿分。

1. The transformation equations for plane polar coordinates (r,θ) may be expressed in the form $\vec{r} = r\cos\theta\,\hat{x} + r\sin\theta\,\hat{y}$ where $\hat{x}$, $\hat{y}$ are constant unit vectors in rectangular coordinates. Write the appropriate expressions for the unit vectors $\hat{r}$ and $\hat{\theta}$ (which are orthogonal) in terms of θ, $\hat{x}$ and $\hat{y}$ and show that $\frac{d\hat{r}}{d\theta} = \hat{\theta}$, and $\frac{d\hat{\theta}}{d\theta} = -\hat{r}$. Using these results, show that the acceleration is

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

where $\dot{r} = \frac{dr}{dt}$, $\dot{\theta} = \frac{d\theta}{dt}$, $\ddot{\theta} = \frac{d^2\theta}{dt^2}$.

Consider now the motion of a particle of mass m in a central-force $\vec{f}(r) = \hat{r} f(r)$, show that the angular momentum of the system is conserved.

Show that the equation of motion may be cast in the suitable form (by making a simple change of variable $u = \frac{1}{r}$)

$$f\left(\frac{1}{u}\right) = -\frac{l^2 u^2}{m}\left(\frac{d^2 u}{d\theta^2} + u\right)$$

wher l is the constant value for the angular momentum. (20%)

2. Discuss the motion of a particle (of mass m) in a central-force field

$$f(r) = -\frac{k}{r^2} - \frac{\alpha}{r^3}, \quad (k > 0, \ \alpha > 0)$$

Show that the motion is described by a precessing ellipse, and determine the angular velocity of precession. (20%)

3. Consider the pair of identical simple pendulums (See Fig. 1) coupled by means of a massless spring, of force constant k. We assume that, when both springs are vertical, the spring is just unstretched. We slect as coordinates x_1 and x_2, measured positively to the right from the equilibrium positions of the two masses. (The length of the string in each case is l, and the mass of the bob in each case is m.) Write the equations of motion for small displacements in terms of the coordinates x_1 and x_2 and other parameters.

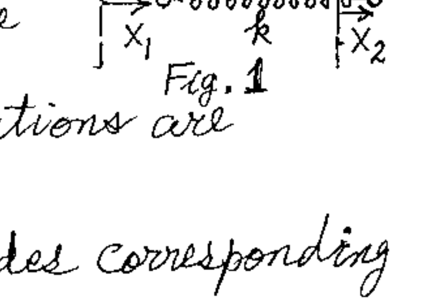

Fig. 1

Show that the eigenfrequencies for small oscillations are

$$\omega_1 = \sqrt{\frac{g}{l}}, \quad \omega_2 = \sqrt{\frac{g}{l} + \frac{2k}{m}}$$

Describe qualitatively the appropriate normall modes corresponding to the above eigenfrequencies. (20%)

4. A simple atomic model (first proposed by Rutherford) may be described briefly as follows. The nucleus is treated as a point charge Ze located at the center of the atom. The electrons are treated as a uniform spherical distribution of charge $-Ze$ $(e > 0)$ concentric with the nucleus and having radius a. Use Gauss's law to

Calculate the electric field $\vec{E}$ at a distance r from the center of this charge distribution for $r > a$ and $r < a$. Using the result just obtained for the electric field to evaluate the potential produced by this charge distribution for both $r < a$ and $r > a$. (20%)

5. A sphere of radius R carries a uniform surface charge density σ over its surface. If the sphere rotates about a diameter with angular velocity ω, show that the magnitude of its magnetic moment is $\mu = \frac{4}{3}\pi R^4 \sigma\omega$. (20%)
If a uniform magnetic field $\vec{B}$ is applied perpendicular to the axis of rotation of the sphere, determine the torque on the magnetic moment of the sphere.

6. Write the Maxwell equations (for electric field $\vec{E}$ and magnetic field $\vec{B}$) of steady charge and current distribution ρ and $\vec{J}$. Show that charge is always conserved. In the absence of the charge and current distribution (i.e. in free space), show that electromagnetic waves can propagate in free space.
Hint: $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}$, and $\vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B}$ etc. (20%)

7. Show that for an ideal gas undergoing an adiabatic process, the state variables P and V satisfy $PV^{\gamma} = \text{constant}$, where $\gamma = C_p/C_V$, C_p is the specific heat at constant pressure, C_V is the specific heat at constant volume. Show also that the work done by the gas (during the adiabatic process) as its volume changes from V_i to V_f is given by

$$W = \frac{p_i V_i}{\gamma - 1}\left[1 - \left(\frac{V_i}{V_f}\right)^{\gamma-1}\right]. \qquad (20\%)$$

8. Using the first law of thermodynamics show that the relation between C_p and C_V for any ideal gas is given by

$$C_p = C_V + R$$

where R is the universal gas constant.
Using the equipartition of energy theorem to evaluate the values of C_p, C_V and γ for (a) monatomic gas (b) rotating vibrating diatomic gases. (20%) (Note that $\gamma = C_p/C_V$)