國立中央大學108學年度碩士班考試入學試題

所別： 天文研究所碩士班 不分組(一般生)　　　　共2頁 第1頁

科目： 普通物理

本科考試禁用計算器　　　　＊計算題需計算過程，無計算過程者不予計分

1. (Total 10%)　A particle moving in x-y plane whose potential energy is

$U = \frac{1}{2}k\left(x^2 + y^2\right)$ and k is a constant

(i) (5%) What is the force acting on the particle?

(ii) (5%) Prove that the angular momentum is conserved for this particle.

2. (Total 30 %) A binary system consists of two stars of masses m_1 and m_2 respectively orbiting around their center of mass in circular orbits. Suppose the separation of two stars is a. Using the center of mass as the origin of the coordinate system,

(i) (10%) Show that the Kepler's 3rd law of this binary system can be written as

$$\frac{a^3}{P^2} = \frac{G(m_1 + m_2)}{4\pi^2}$$

where P is the orbital period of the binary system and G is the gravitational constant.

(ii) (5%) What is the total energy E of the system? Write it down as a function of m_1, m_2, and a.

(iii) (15%) The total energy of the binary system would be lost due to emission of gravitational wave with a rate of

$$\frac{dE}{dt} = -\frac{32G^4}{5c^5}\frac{(m_1 + m_2)m_1^2 m_2^2}{a^5}$$

Suppose the energy loss rate is very small so the orbit is still circular and the Kepler's 3rd law is still hold during the evolution. What is $\frac{dP}{dt}$? Write it down as a function of m_1, m_2 and P

3. (Total 20%)There is a system of the monoatomic ideal gas with fixed molecular number N in a container

(i) (5%) Please derive the entropy S of as the function of its internal energy U and volume V, that is $S(U,V)$

(ii) (5%) What is the final temperature T_2 for the system processing adiabatic expansion　(絕熱膨脹) from V_1 to V_2 with the initial temperature of T_1

(iii) (5%) What is the entropy change for the system processing isothermal expansion

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(等溫膨脹) from V_1 to V_2

(iv) (5%) What is the entropy change for the system processing isobar expansion (等壓膨脹) from V_1 to V_2

4. (Total 15%) There is a sphere of radius R with spherical symmetry charge density $\rho(r) = \rho_c\left(1 - \frac{r}{R}\right)$ where ρ_c is the charge density at the center.

(i) (5%) What is the total charge in this sphere?

(ii) (10%) Derive the electric fields and potential for $r < R$ and $r > R$ if we define $\Phi(r \to \infty) = 0$

5. (10%) An antiproton can be made by colliding two protons as $p + p \to p + p + p + \bar{p}$ where p is proton and $\bar{p}$ is antiproton. Suppose initially one of the protons (proton 1) is rest, what is the minimum kinetic for another proton (proton 2) to make this reaction happen if the rest mass of proton is M_p.

6. (Total 15%) In quantum mechanics, the parity operator $\hat{P}$ is the flip in the sign of spatial coordinate, that is, $x \to -x$ in one dimensional system. For example, for any function $\varphi(x)$, $\hat{P}\varphi(x) = \varphi(-x)$

(i) (5%) Show that the only possible eigenvalues of $\hat{P}$ is either 1 or -1.

(ii) (10%) Suppose $u_E(x)$ is the eigenfunction of Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + V(x)$ with eigenenergy of E where $\hat{p}$ is momentum operator, show that $\hat{P}u_E(x)$ is also an eigenfunction of $\hat{H}$ with same eigenenergy if the potential $V(x)$ is an even function

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