國立中央大學103學年度碩士班考試入學試題卷

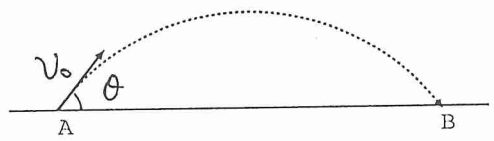
所別：天文研究所碩士班 不分組(一般生)　科目：普通物理　共 2 頁 第 1 頁
天文研究所碩士班 不分組(在職生)

本科考試禁用計算器　　＊請在試卷答案卷（卡）內作答

1 Trajectory of a ball

A ball is thrown from a point A at an angle θ. The initial velocity of the ball is v_0, and the ball touches the ground at a point B. Consider the trajectory of this ball in 2-dimentional space. The resistance of the air is ignored.

1. Consider the force acting on the ball, and show the equations of motion. Describe your choice of the coordinate axes and the origin of the time. (10 points)
2. Integrate the equations of motion, and derive the position of the ball as a function of time. (10 points)
3. Derive the maximum height of the ball. (5 points)
4. What is the distance AB? (5 points)



2 Snell's law

Consider the refraction of light. The light travels from the medium of refractive index n_1 into the medium of refractive index n_2.

The optical path length w can be calculated by

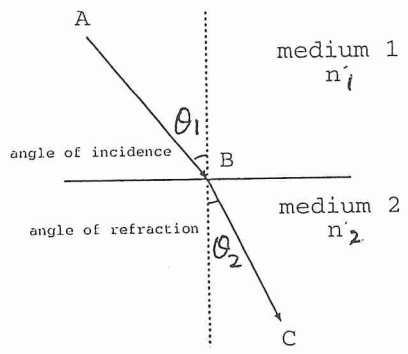
$$w = \int n ds, \tag{1}$$

where n is the refractive index of the medium and ds is the distance that the light travels.

Consider the optical path length of $AB + BC$, and choose the route that minimizes the optical path length. Then, show Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{2}$$

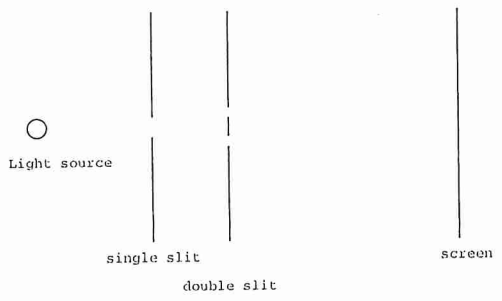
where θ_1 is the angle of incidence and θ_2 is the angle of refraction. (10 points)



3 Young's double-slit experiment

The figure shows the configuration of Young's double-slit experiment. Bright and dark bands are observed on the screen.

1. Describe why we see bright and dark bands. (10 points)
2. Calculate the locations of bright and dark bands. Mention clearly the coordinate you choose. (10 points)



注意：背面有試題

4 Properties of ideal gas

1. Consider a box of the size $L \times L \times L$ filled with gas. The macroscopic pressure of the gas can be understood by the sum of momentum changes of gas particles when they hit the internal wall of the box. A particle with mass m_g is flying with the velocity v. What is the amount of the momentum given to the wall by a single collision of a gas particle to the wall? Assume the elastic collision from the normal direction. (5 points)

2. There are N particles in total in the box, and the mean velocity of gas particles is $\bar{v}$. Show that the pressure of the gas P can be expressed as

$$P = \frac{N m_g \bar{v}^2}{3V}, \tag{3}$$

where V is the volume of the box. (5 points)

3. The molar specific heat at constant pressure C_P is defined as

$$nC_P = \left(\frac{\partial Q}{\partial T}\right)_P, \tag{4}$$

where Q is the net energy transfered to the system, T is the temperature, and n is the number of moles. The molar specific heat at constant volume C_V is defined as

$$nC_V = \left(\frac{\partial Q}{\partial T}\right)_V. \tag{5}$$

Use the first law of thermodynamics and the equation of state to derive Mayer's relation

$$C_P - C_V = R, \tag{6}$$

where R is the gas constant. Note that the internal energy of ideal gas depends only on the temperature. (10 points)

4. For adiabatic processes, show the relation

$$PV^\gamma = \text{constant}, \tag{7}$$

where $\gamma = \dfrac{C_P}{C_V}$. (10 points)

5 Electric dipole

The electrostatic potential ϕ by a point charge q at a distance r can be expressed as

$$\phi = \frac{q}{4\pi\varepsilon_0 r}. \tag{8}$$

Consider two electric point charges apart a distance d on z-axis. Electric charges of these two are $+q$ and $-q$, respectively. Show that the electrostatic potential due to this dipole at a position P is

$$\phi = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}, \tag{9}$$

where r is OP distance and θ is the angle between z-axis and OP. Assume $r \gg d$. (10 points)