國立中央大學101學年度碩士班考試入學試題卷

所別：天文研究所碩士班 不分組(一般生)　科目：普通物理　共 2 頁 第 1 頁
天文研究所碩士班 不分組(在職生)

本科考試禁用計算器　＊請在試卷答案卷（卡）內作答

[I] We consider central core of evolved star. In massive star case, the star will generate core-collapse supernova.

(1) We consider the central core that have mass of M =3.0 x 10^{30} kg. This core will have gravitational contraction, then change the radius from $R_1=10^6$m to $R_2=10^4$m. Please estimate the released gravitational energy ΔU (just equation). Here, we assume the density of the central core is constant. You can use gravitational constant as G (6.67×10^{-11} m^3 kg^{-1} s^{-2}). [7 points]

(2) Now, we consider the supernova explosion occurred at the Galactic Center (the distance from Earth is L =3×10^{20}m). Please estimate the observed number of neutrinos per unit area at Earth. Here, we assume all released gravitational energy (estimate in [1]) changes to neutrinos and neutrinos are released isotropically. The averaged neutrino energy is E_ν=10MeV. [8 points]

(3) One-sixth of the neutrino is electron antineutrino ν_e. An instrument will detect ν_e with following reaction.

$$\nu_e + p \rightarrow n + e^+$$

Now, we will use the instrument which weight is $M_D = 3\times10^6$ kg. Please estimate the number of neutrinos with this instrument (both equation and actual number). The cross section of the reaction σ is 10^{-46} m^2. The proton ratio in the instrument is ½. [10 points]

[II] We consider the Blackbody radiation from the astronomical object that has temperature of T. The intensity $I_T(\nu)$ is given by the Planck's law.

$$I_T(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{k_B T}\right)-1}$$

Here, *ν, h, c and* k_B are frequency of electro-magnetic wave, the Planck constant, light speed and Boltzmann constant.

(1) At the $h\nu/k_B$T << 1 limit, $I_T(\nu)$ is proportional to ν and T. Please estimate the indexes, respectively. [5 points]

(2) $I_T(\nu)$ has the peak at $\nu=\nu_{peak}$(T). Please prove the ν_{peak}(T) is proportional to T. Please also derive following equation when we assume $\nu_{peak}(T) = Ak_BT/h$. Here, A is a constant. [5 points]

(3) Please draw the shape of the $I_T(\nu)$ for T=10^4[K] and 10^7[K] on the double logarithmic chart(X axis : ν, Y axis : $I_T(\nu)/I0$). Here, I0 is the spectral peak at the T=10^4[K]. [5 points]

(4) Please estimate ν_{peak} when A=3, $k_B/h = 2 \times 10^{10}$ [Hz K^{-1}]. Please also classify the electro-magnetic wave around this peak into radio, infrared, optical, ultra-violet, X-ray or γ -ray. [5 points]

(5) Now, we define F(T) as the total energy per unit area per unit time from the blackbody that temperate is T. Please prove that the F(T) is proportional to the fourth power of T (F(T) ∝ T^4). [5 points]

注意：背面有試題

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[III]

(1) Please fill up following table. (The Planck constant $h = 4 \times 10^{-15}$ eV sec, the speed of light $c = 3 \times 10^{8}$ m/s.)

	Wavelength	Frequency	Energy	Name of light
A		80 MHz		
B	500 nm			
C			100 keV	

[9 points]

(2) Please list up light sources around B as much as possible. [6 points]

(3) Now, we consider a nuclear that has $+Ze$ charge and an electron has circular motion around the nuclear. We assume that the atom is stable when the angular momentum of the electron take whole-number multiple of $h/2\pi$.
Please prove that the acceptable electron energy is $E_n = -\frac{2\pi^2 m Z^2 e^4}{n^2 h^2}$. [5 points]

(4) To ionize Hydrogen, what kind of light do we need to radiate? If needed, please use m=9.1x10^{-31} kg, e = -1.6x10^{-19} C. [5 points]

[IV] We consider motion of particle in a limited space.
There is a law as $-\langle k \rangle = -\langle \sum_{i=1}^{N} \frac{P_i}{2m_i} \rangle = \frac{1}{2} \sum_i^N \langle F_i \cdot r_i \rangle$.
k: Kinematic energy,
<...> : long term time average,
P_i : Momentum of particle,
m_i: mass of particle,
F_i : Force to the particle.

Please prove n<U> =2<K> when the force can be written $F_i = -\nabla_{ri} U$ ($U \propto r^n$). [2 points]
Please also derive the quation for gravit and electromagnetic force cases. [3 points]

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