

國立中央大學101學年度碩士班考試入學試題卷

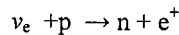
所別：天文研究所碩士班 不分組(一般生) 科目：普通物理 共 2 頁 第 / 頁
天文研究所碩士班 不分組(在職生)

本科考試禁用計算器

*請在試卷答案卷(卡)內作答

[I] We consider central core of evolved star. In massive star case, the star will generate core-collapse supernova.

- (1) We consider the central core that have mass of $M = 3.0 \times 10^{30}$ kg. This core will have gravitational contraction, then change the radius from $R_1 = 10^6$ m to $R_2 = 10^4$ m. Please estimate the released gravitational energy ΔU (just equation). Here, we assume the density of the central core is constant. You can use gravitational constant as $G (6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})$. [7 points]
- (2) Now, we consider the supernova explosion occurred at the Galactic Center (the distance from Earth is $L = 3 \times 10^{20}$ m). Please estimate the observed number of neutrinos per unit area at Earth. Here, we assume all released gravitational energy (estimate in [1]) changes to neutrinos and neutrinos are released isotropically. The averaged neutrino energy is $E_\nu = 10 \text{ MeV}$. [8 points]
- (3) One-sixth of the neutrino is electron antineutrino $\bar{\nu}_e$. An instrument will detect $\bar{\nu}_e$ with following reaction.



Now, we will use the instrument which weight is $M_D = 3 \times 10^6$ kg. Please estimate the number of neutrinos with this instrument (both equation and actual number). The cross section of the reaction σ is 10^{-46} m^2 . The proton ratio in the instrument is $\frac{1}{2}$. [10 points]

[II] We consider the Blackbody radiation from the astronomical object that has temperature of T . The intensity $I_T(\nu)$ is given by the Planck's law.

$$I_T(\nu) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

Here, ν , h , c and k_B are frequency of electro-magnetic wave, the Planck constant, light speed and Boltzmann constant.

- (1) At the $h\nu/k_B T \ll 1$ limit, $I_T(\nu)$ is proportional to ν and T . Please estimate the indexes, respectively. [5 points]
- (2) $I_T(\nu)$ has the peak at $\nu = \nu_{\text{peak}}(T)$. Please prove the $\nu_{\text{peak}}(T)$ is proportional to T . Please also derive following equation when we assume $\nu_{\text{peak}}(T) = A k_B T / h$. Here, A is a constant. [5 points]
- (3) Please draw the shape of the $I_T(\nu)$ for $T = 10^4 [\text{K}]$ and $10^7 [\text{K}]$ on the double logarithmic chart (X axis : ν , Y axis : $I_T(\nu)/I_0$). Here, I_0 is the spectral peak at the $T = 10^4 [\text{K}]$. [5 points]
- (4) Please estimate ν_{peak} when $A = 3$, $k_B/h = 2 \times 10^{10} [\text{Hz K}^{-1}]$. Please also classify the electro-magnetic wave around this peak into radio, infrared, optical, ultra-violet, X-ray or γ -ray. [5 points]
- (5) Now, we define $F(T)$ as the total energy per unit area per unit time from the blackbody that temperate is T . Please prove that the $F(T)$ is proportional to the fourth power of T ($F(T) \propto T^4$). [5 points]

注：背面有試題

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[III]

(1) Please fill up following table. (The Planck constant $h = 4 \times 10^{-15}$ eV sec, the speed of light $c = 3 \times 10^8$ m/s.)

	Wavelength	Frequency	Energy	Name of light
A		80 MHz		
B	500 nm			
C			100 keV	

[9 points]

(2) Please list up light sources around B as much as possible.

[6 points]

(3) Now, we consider a nuclear that has $+Ze$ charge and an electron has circular motion around the nuclear. We assume that the atom is stable when the angular momentum of the electron take whole-number multiple of $h/2\pi$.

Please prove that the acceptable electron energy is $E_n = -\frac{2\pi^2 m Z^2 e^4}{n^2 h^2}$.

[5 points]

(4) To ionize Hydrogen, what kind of light do we need to radiate? If needed, please use $m=9.1 \times 10^{-31}$ kg, $e = -1.6 \times 10^{-19}$ C.

[5 points]

[IV] We consider motion of particle in a limited space.

There is a law as $-\langle k \rangle = -\langle \sum_{i=1}^N \frac{P_i}{2m_i} \rangle = \frac{1}{2} \sum_{i=1}^N \langle F_i \cdot r_i \rangle$.

k : Kinematic energy,

$\langle \dots \rangle$: long term time average,

P_i : Momentum of particle,

m_i : mass of particle,

F_i : Force to the particle.

Please prove $n\langle U \rangle = 2\langle K \rangle$ when the force can be written $F_i = -\nabla_{r_i} U$ ($U \propto r^n$).

[2 points]

Please also derive the quation for gravit and electromagnetic force cases.

[3 points]

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