科目：工程數學A(5002)

參考用

校系所組：中大光電科學與工程學系、照明與顯示科技研究所
清大電機工程學系甲組、光電工程研究所
清大電子工程研究所、工程與系統科學系丁組
清大動力機械工程學系乙組
陽明醫學工程研究所醫學電子組、
陽明生醫光電工程研究所理工組B

1. Consider the ODE $(3y^2+x+1)dx+2y(x+1)dy=0$.

(1) (4%) Find an integrating factor for the ODE.

(2) (4%) Given $y(0)=1$, solve the initial value problem.

2. Consider a mass-spring system governed by the ODE $y''+6y'+18y=-90\sin(6t)$.

(1) (3%) How would you describe this system (choose one below)?
(A) Undamped; (B) Underdamped; (C) Critical damped; (D) Overdamped.
(2) (5%) Find the steady-state solution.

3. Consider the ODE $x^3y'''+8x^2y''+9xy'-9y=0$ for $x>0$.

(1) (5%) Find a basis of solutions $\{y_1(x), y_2(x), y_3(x)\}$ for the ODE.

(2) (4%) Given initial conditions $y(1)=0$, $y'(1)=-2$, and $y''(1)=2$, solve the initial value problem.

4. (5%) Bessel function of the first kind of order ν, $J_\nu(x)$, is one solution of the Bessel equation, $x^2y''+xy'+(x^2-\nu^2)y=0$. The general solution of the ODE, $x^2y''+xy'+(4x^4-\frac{1}{9})y=0$, can be expressed as $y(x)=C_1J_\nu\left(ax^2\right)+C_2J_{-\nu}\left(ax^2\right)$. Determine the values of a and ν.

5. (10 %) Use Laplace transform to solve $xy''+(1-x)y'+ky=0$.

(A) $y=\frac{e^t}{k!}\frac{d^k}{dt^k}[t^ke^{-t}]$ (B) $y=\frac{e^t}{k}\frac{d^k}{dt^k}[t^ke^t]$ (C) $y=\frac{e^t}{k}\frac{d^k}{dt^k}[t^ke^{-t}]$ (D) $y=\frac{e^t}{k!}\frac{d^k}{dt^k}[t^{-k}e^{-t}]$ (E) $y=\frac{e^{-t}}{k!}\frac{d^k}{dt^k}[t^ke^{-t}]$
(F) $y=\frac{e^{-t}}{k}\frac{d^k}{dt^k}[t^ke^{-t}]$ (G) $y=\frac{e^k}{t!}\frac{d^k}{dt^k}[t^ke^{-t}]$ (H) none of the above

6. (10 %) Find the Fourier transform of $f(x)=\sqrt{\frac{\pi}{2}}$ if $|x|<2$ and $f(x)=0$ otherwise.

(A) $f(w)=\frac{\sin w}{w}$ (B) $f(w)=\frac{\sin w}{2w}$ (C) $f(w)=\frac{\cos w}{w}$ (D) $f(w)=\frac{\cos w}{2w}$
(E) $f(w)=\sqrt{\frac{\pi}{2}}\frac{\sin w}{w}$ (F) $f(w)=\sqrt{\frac{2}{\pi}}\frac{\sin w}{w}$ (G) $f(w)=\frac{\cos 2w}{w}$ (H) none of the above

注意：背面有試題

7. Consider the problem

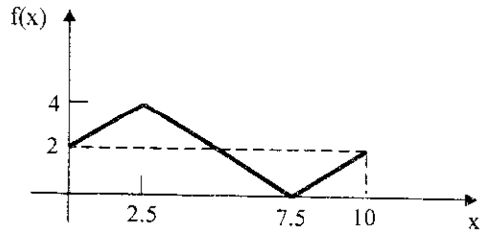
$u_{tt} - 4u_{xx} = 0 \qquad 0 < x < 10$

$u(0,t) = u(10,t) = 2 \qquad 0 < t$

$u(x,0) = f(x) \qquad 0 < x < 10$

$u_t(x, t=0) = 0 \qquad 0 < x < 10$

$f(x)$ is shown in the following figure.



(1) (7%) What is $u(2,1)$ (the value of u at position x = 2 when t = 1)?

(A) 0.8 (B) 1.2 (C) 1.6 (D) 2 (E) 2.4 (F) 2.8 (G) 3.2
(H) none of the above.

(2) (6%) What is the lowest frequency (cycles per time) of the motion of u?

(A) 0.05 (B) 0.1 (C) 0.2 (D) 0.4 (E) 0.8 (F) 1.6 (G) 3.2
(H) none of the above.

8. (7%) The temperature distribution of a thin bar is described by a 1-D heat equation

$u_t - 4u_{xx} = 0 \qquad 0 < x < 10$

The boundary and initial conditions are given as follows:

$u(0,t) = u(10,t) = 0 \qquad 0 < t$

$u(x,0) = \sin\frac{\pi x}{10} \qquad 0 < x < 10$

The peak temperature is located at the position x = 5 at all time. At what time will the peak temperature reduce to 1/e of its initial value?

(A) $\pi/10$ (B) $\pi^2/100$ (C) $10/\pi$ (D) $100/\pi^2$ (E) $\pi/5$
(F) $\pi^2/25$ (G) $5/\pi$ (H) none of the above.

9. (20%) Evaluate the principal value of the integral $\int_{-\infty}^{\infty} \frac{\cos 3x}{x^3 + x^2 + 3x - 5}\,dx$.

10. (10%) Find the eigenvalues and corresponding normalized eigenvectors (norm equals to 1) for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$. What are those for the transpose matrix A^T?