

1. Find the solution satisfying the given condition: $y' = \frac{1}{x^2}y^2 - \frac{1}{x}y + 1$; $y(1)=3$.
(10%)

2. Solve the initial value problem: $y'' + 3y' - 2y = 0$; $y(0) = 2$, $y'(0) = -3$ (10%)

3. Solve the initial value problem using Laplace transforms:

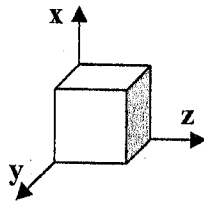
$$y^{(4)} - 2y'' + y = 1; \quad y(0) = y'(0) = y''(0) = y^{(3)}(0) = 0 \quad (10\%)$$

4. Solve the general solution of $y'' + \left(\frac{81}{4}x - \frac{35}{4x^2}\right)y = 0$ (10%)

5. Find the inverse of the matrix $A = \begin{bmatrix} 8 & 0 & 1 \\ 3 & -2 & 1 \\ 1 & 4 & 0 \end{bmatrix}$ (10%)

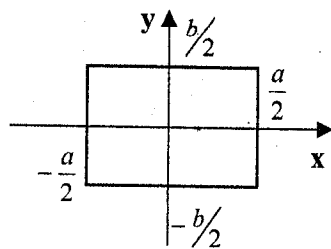
6. Evaluate the integral: $\int_0^{\infty} \frac{\sqrt{x} dx}{1+x^2}$ (12%)

7. Evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{A}$ over a unit cube defined by the point (0,0,0) and the unit intercepts on the positive x-, y-, and z-axes (see figure below). (8%)



8. Find the complex form of the Fourier series of the following periodic functions of period 2π :
 $f(x) = x$; $(-\pi < x < \pi)$. (10%)

9. Find the eigenfunctions of the rectangular membrane (size $a \times b$) shown below. (10%)



10. y is a function of x . Please find the solution for the differential equation $\frac{d^2y}{dx^2} = e^y$. (10%)