國立中央大學九十一學年度碩士班研究生入學試題卷

所別：光電科學研究所　不分組　科目：應用數學　共 1 頁 第 1 頁

(1) State the definition of the following 6 kinds of matrices:

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i. symmetric matrix
ii. skew-symmetric matrix
iii. Hermitian matrix
iv. skew-Hermitian matrix
v. orthogonal matrix
vi. unitary matrix

(2) State(寫出，不必證明) the characteristics of the eigenvalues and eigenvectors of the 6 kinds of matrices in problem 1. These characteristics include at least:

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i. Are these eigenvalues real or imaginary or complex? Do they possess any other features?
ii. Is there any relationship between the different eigenvectors of each kind these matrices? For example, are they orthogonal to each other?

(3) a) Given a vector $\vec{a} = \hat{e}_1 + 2\hat{e}_2 + 3\hat{e}_3$, where $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ are the 3 unit vectors along the x-, y-, and z-axis. Find the equation for a plane normal to this vector and passing the origin of the coordinate system.

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b) Find the equation for the plane normal to this vector and passing the point (0,1,0).

(4) Given $f(x) = \frac{2}{L}x + \frac{1}{2}$ when $0 < x < \frac{L}{2}$

$= -\frac{4}{L}x + 4$ when $\frac{L}{2} < x < L$

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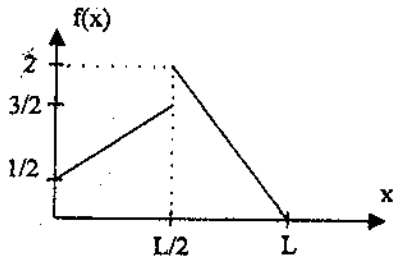
a) Find the half range Fourier expansion with odd periodic continuation of their function.



b) Draw a figure of your obtained series, including several cycles.

c) Does your obtained series really represent the given function at every point between 0 and L inclusively, give comments (試討論之).

(5) a) Find $\frac{d}{dx}x^x$ at x = e, where e is the base of national logarithm.

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b) Find $\frac{d}{dz}z^{(3+4i)}$ at $z = 3 + 4i$

(6) (10%) 請解 $(x^2+1)y' + y^2 + 1 = 0, \quad y(0) = 1$

(7) (10%) 在何種條件下，方程式為 exact？並解其方程式。

$(x^3 + xy^2)dx + (ax^2y + h(x,y))dy = 0$

(8) (10%) 請解 $y'' + 4y' + (4+\omega^2)y = 0, \quad y(0) = 1, \quad y'(0) = \omega - 2$

(9) (10%) 請解 Partial Equation $\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$

參考用