

中央大學八十九學年度碩士班研究生入學試題卷

光電科學研究所 不分組 科目:

應用數學

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1. Find the solution of the following equations with given initial conditions:

$$\frac{dy_1}{dt} = 3y_1 + 4y_2 \quad \frac{dy_2}{dt} = 4y_1 - 3y_2$$

Initial conditions: $y_1(0) = 1, y_2(0) = 3$ (14%)

2. Evaluate the following series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots$$

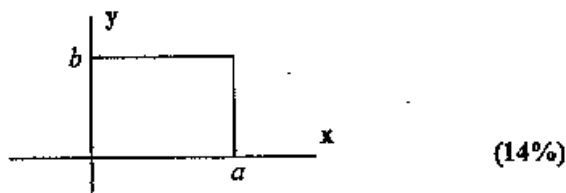
by using the result of Fourier series expansion for the following function which is assumed to have a period 2π :

$$f(x) = \begin{cases} 1 & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases} \quad (14\%)$$

3. Evaluate the following integral:

$$\int_0^{2\pi} \frac{d\theta}{5 - 4 \cos \theta} \quad (14\%)$$

4. Find the eigenfunctions of a vibrating rectangular membrane (size $a \times b$) which is fixed at the boundary.



5. For a matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ \lambda & \lambda^2 & 1 \end{bmatrix},$$

(a) what should λ be in order for the inverse matrix to exist?

(b) For $\lambda = 1$, find the inverse matrix of the matrix A. (14%)

6. Evaluate the following line integral:

$$\int_{(3,0,0)}^{(2,3,4)} (x dx + y dy + z dz) \quad (10\%)$$

7. Find the distance between the point $(1, 1, 2)$ and the plane which is defined by three points $(0, 1, 0)$, $(1, 1, 3)$, and $(5, 0, 1)$. (10%)

8. Find the directional derivative of f at P in the direction of a

$$f = e^x \cos y, \quad P = (2, \pi, 0), \quad a = 2\hat{i} + 3\hat{k} \quad (10\%)$$