1. Consider $f(\rho)=\frac{e^{jk\rho}}{\rho}$ (see Fig. 1). Find the derivative of $f(\rho)$ in the direction of $\hat{r}_0$ at point P. The circle in the figure represents a sphere with center at O; r_o, being the radius of the sphere, is *10 cm*; $k=\frac{2\pi}{0.6\times10^{-4}\,cm}$; j is the unit imaginary number. You may use any approximation in your final answer. Given $\tan^{-1}\frac{1}{2}=26.6^\circ$. (12%)

2. Let $f(z)=\cos\bar{z}$, where $z=x+iy$, $\bar{z}=x-iy$,

(5%) a) Is $f(z)$ continuous everywhere in the xy plane. Explain.

(7%) b) Does it possess a derivative with respect to z everywhere in the xy plane. If it possess a derivative with respect to z, find it.

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3. It is known that

(11%) $Ln(1+z)=z-\frac{z^2}{2}+\frac{z^3}{3}-\frac{z^4}{4}+\ldots$

where $Ln(1+z)$ is the principal value of the complex logarithmic function $ln(1+z)$. Find the radius of convergence of the above series.

4. a) Find the points at which the complex valued function Ln z is not analytic.

(4%) b) Expand the function in a Tayor series centered at $-1+i$.

(7%)

(4%) c) The radius of convergence of this series is $\sqrt{2}$. Suppose you are asked to find $Ln(-1-0.2i)$, which series are you going to use for this calculation? The series in this problem or the series in problem 3. Why?

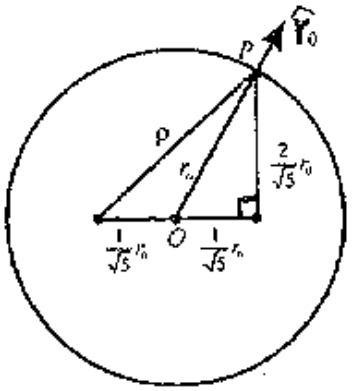

Fig. 1

5. (14%) Find the solution of y(x)

(a) $xy'' + 2(1-x)y' + (x-2)y = 2e^x$.

(b) $x^3y' + 3x^2y + x^2 - 1 = 0, \quad y(1) = 1$.

6. (12%) Find A(x) and B(x)

$$\frac{\partial A}{\partial x} = -iBe^{-ix}$$

$$\frac{\partial B}{\partial x} = -iAe^{ix}, \quad A(0)=1,\ B(0)=0.$$

7. (a) (8%) Find the eigenvalues and the orthogonal eigenvectors of the matrix A

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{bmatrix}$$

(b) (4%) Find the eigenvalues of A^4

8 (12%)As shown in the figure, the center of the sphere is located at O (0,0,0). There are two points A $(\frac{3}{4}, -\frac{\sqrt{3}}{4}, \frac{1}{2})$ and B $(\frac{1}{4}, \frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2})$ on the sphere. C is the middle point between A and B. Now we rotate the coordinate system so that the new x axis is along $\overrightarrow{OC}$ and the new y axis is parallel to $\overrightarrow{AB}$. What are the new coordinates of T (0,0,1).