國立中央大學八十五學年度碩士班研究生入學試題卷

所別: 光電科學研究所　不分組　科目: 應用數學　共 1 頁 第 1 頁

1. Find the general solution of the following equation:
$y'' + (y')^2 + 1 = 0$. (10%)

2. Find the directional derivative of the function $f = e^x \cos y$ at the position $(2, \pi, 0)$ in the direction $\vec{a} = 2\hat{i} + 3\hat{k}$. (10%)

3. Evaluate
$$I = \int_{(1,1,2)}^{(3,-2,-1)} (yzdx + xzdy + xydz) \ . \quad (10\%)$$

4. Compute $f(a,b) = \int_0^\infty \frac{dx}{x}(e^{-ax} - e^{-bx})$. (12%)

5. Evaluate the following integral by contour integration
$$I = \int_0^{2\pi} \frac{d\theta}{2 + \cos\theta} \ . \quad (13\%)$$

6. Find the inverse matrix of the symmetric matrix C which satisfies the equation $Q = X^T C X$, where $X^T = (x_1 \ x_2 \ x_3)$ and $Q = x_1^2 + 2x_1x_2 + 3x_2^2 + 6x_2x_3 + 2x_3^2$. (15%)

7. Using Laplace transform to solve the equation
$y''' + y' = t$, where $y(0) = 0, \ y'(0) = 1, \ and \ y''(0) = 0$.
(15%)

8. Find the Fourier series expansion of the periodic function f(x) with period 2π, where $f(x) = 0$ *for* $-\pi < x < 0$ and $f(x) = \sin(x)$ *for* $0 < x < \pi$. From the result find the sum
$$S = \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \frac{1}{9\cdot11} + \cdots \ . \quad (15\%)$$