國立中央大學 112 學年度碩士班考試入學試題

所別： 光電類　　共 2 頁　第 1 頁

科目： 工程數學

Choose the correct answer (50 %): 單選題，每題 5 分
（單選題請在答案卷上作答）

(1) Consider a photon beam of intensity $\mathbf{I_0}$, if the beam contains only photons of a single wavelenth λ and is directed at an absorptive semiconductor of thickness L, we can assume that the degradation of the intensity -d**I**(x)/dx is proportional to **I**(x), where **I**(x) is the intensity remaining at x. For $\mathbf{I_0}$ = 10 mW, λ = 400 nm and L = 0.46 μm, what is the relationship between -d**I**(x)/dx and **I**(x) if 1 mW is absorbed by the semiconducor?
(A) $-d\mathbf{I}(x)/dx = a^2\,\mathbf{I}(x)$, with $a = 1\times10^4$ cm^{-1}; (B) $-d\mathbf{I}(x)/dx = a^3\,\mathbf{I}(x)$, with $a = 1\times10^4$ cm^{-1};
(C) $-d\mathbf{I}(x)/dx = a\,\mathbf{I}(x)$, with $a = 2\times10^4$ cm^{-1}; (D) $-d\mathbf{I}(x)/dx = a^2\,\mathbf{I}(x)$, with $a = 2\times10^4$ cm^{-1};
(E) $-d\mathbf{I}(x)/dx = a^3\,\mathbf{I}(x)$, with $a = 3\times10^4$ cm^{-1}; (F) $-d\mathbf{I}(x)/dx = a\,\mathbf{I}(x)$, with $a = 4\times10^4$ cm^{-1};
(G) $-d\mathbf{I}(x)/dx = a\,\mathbf{I}(x)$, with $a = 5\times10^4$ cm^{-1}.

(2) If c is a constant and $x\frac{dy}{dx} = y + e^x y^3$, with $0 < x < \infty$, what is y(x)? (A) $(2x^{-2}-cx^{-1}e^x+2x^{-2}e^x)^{-0.5}$
(B) $(x^{-2}-2x^{-1}e^x+2x^{-2}e^c)^{-0.5}$ (C) $(cx^{-2}-2x^{-1}e^x+2x^{-2}e^x)^{-0.5}$ (D) $(cx^{-2}-3x^{-1}e^x+2x^{-2}e^x)^{-0.5}$
(E) $(3cx^{-2}-2x^{-1}e^x+2x^{-2}e^x)^{-0.5}$ (F) $(x^{-2}-2x^{-1}e^x+2x^{-2}e^x+c)^{-0.5}$ (G) Nove of the above

(3) What is the solution of $\frac{dy}{dx} = \frac{2+sinx}{3(y-1)^2}$ with $y(0)=1$? (A) $x = 1 + (2y - cosy + 1)^{1/3}$
(B) $y = 1 + (2x - cosx + 1)^{1/3}$ (C) $y = 1 + (2x - cosx + 2)^{1/3}$ (D) $y = 1 + (2x - cosx + 3)^{1/3}$
(E) $x = 1 + (2y - cosy + 2)^{1/3}$ (F) $x = 1 + (2y - cosy + 3)^{1/3}$ (G) None of the above

(4) If $\frac{dR}{dt} = 2R$ and $R = -1$ when $t = 0$, what is R when $t = 1$? (A)7.389 (B) -5.389 (C) 8.389
(D) -3.389 (E) 6.389 (F) -2.389 (G) None of the above.

(5) ***T*** is a radioactive material with the half-life of 1.4×10^{10} years. If a piece of mineral contains 2 grams of ***T***, how many years must elapse before1.9 grams of ***T*** reamin in the mineral? (A) 0.104×10^{10}
(B) 0.204×10^{10} (C) 0.304×10^{10} (D) 0.404×10^{10} (E) 0.504×10^{10} (F) 0.604×10^{10} (G) 0.704×10^{10}

(6) A bacterium is cultivated in a beaker of liquid. If P(t) is the population density of the bacterium at the t^{th} hour and P(t) triples in 2 hours, how many hours will P(t) double? (A) 1.062 (B) 1.162
(C) 1.262 (D) 1.362 (E) 1.4262 (F) 1.562 (G) 1.662

(7) If y satisfies the equation $x^2\frac{dy}{dx} = y^2 - xy + x^2$, with $y(1) = 2$, what is $y(2)$? (A) 5.518
(B) 6.518 (C) 7.518 (D) 8.518 (E) 9.518 (F) 10.518 (G) 11.518

(8) What is the solution of $(2x + 2y^2) + (4xy + 3y^2)\frac{dy}{dx} = 0$? (A) $x^2+2xy+y^3$ = constant
(B) $x^2+2xy^2+y^3$ = constant (C) $x^2+2xy^2+y^2$ = constant (D) $x+2xy^2+y^3$ = constant
(E) $x+2x^2y^2+y^3$ = constant (F) x^2+2xy^2+y = constant (G) $x+2xy^2+y$ = constant

注意:背面有試題

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(9) If $(2x + 1 + 2y^2) + (4xy + 3y^2)\frac{dy}{dx} = 0$ and $y(0) = -1$, what is x when $y = 1$? (A) 3 (B) -0.5 (C) 2 (D) -2 (E) 0 (F) 1 (G) 0.5

(10) If $2xy = (4y^2 + xy)\frac{dy}{dx}$, what is $y(3)$? (A) -3 (B) 2 (C) 1 (D) 0 (E) -2 (F) 3 (G) None of the above

Solve the following problems (50 %): 計算題(無計算過程者不予計分)

(11) (10%) Consider the vector function $\vec{F}(x, y, z) = [x - y, y - z, z - x]$ and the surface S bounding the hemisphere $x^2 + y^2 + z^2 \le 9,\ z \ge 0$ together with the disk $x^2 + y^2 \le 9$ in the xy-plane. Evaluate $\iint_S \vec{F} \cdot \vec{n} dA$.

(12) Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 3 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

(a) (10%) An orthogonal matrix **X** would diagonalize **A** to a diagonal matrix **D**. Find **X** and **D**.

(b) (3%) Find $\mathbf{A}^{-1}$, the inverse of **A**.

(c) (2%) Find an orthogonal matrix **Y** that diagonalizes $\mathbf{A}^{-1}$ to a diagonal matrix **E**.

(13) (10%) Consider a single-loop RC-series circuit with $R = 10\ \text{k}\Omega$ and $C = 10\ \mu\text{F}$. Determine the current $i(t)$ with an impressed voltage $v(t) = 5\ \text{V}$ for $1 < t < 3$ and zero otherwise. Assume zero initial current. Find the solution by using the Laplace transform.

(14) Denote the Fourier transform of $f(t)$ as $F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$.

(a) (10%) Determine the Fourier transform of $f_1(t) = \cos(2t) \exp(-3|t|)$.

(b) (5%) Find and sketch the Fourier transform of $f_2(t) = \delta(t + \frac{1}{2}) - \delta(t - \frac{1}{2})$, where $\delta(t)$ is the Dirac delta function.

注意:背面有試題