*Boltzmann constant* $k=8.617\times10^{-5}$ eV/K

*Electron mass* $m_e=0.511$ MeV/c$^2$

*Planck's constant* $h=4.136\times10^{-15}$ eV · s

*計算題需計算過程，無計算過程者不予計分

1. (10 pts) A star which is 4.3 light-years away from earth. A rocket leaves for the star at a speed of v = 0.95c relative to the Earth. Assume that the Earth and the star are stationary with respect to one another. According to the astronauts, how much did they age (in years) during their journey?

2. (10 pts) Consider the following set of ions: $H^+$, $He^{2+}$, $Li^{2+}$ and $B^{3+}$. To which ones can we apply Bohr's quantized energy formula? Explain your reasoning.

3. (10 pts) Consider a particle in a box (a.k.a. the infinite square well) with a length of *L*. Calculate the probability that a particle, in the ground state, will be found within the position from $L/4$ L to $3L/4$.

4. (10 pts) The 2nd bound state (n = 1) wave function of the harmonic oscillator is given by:

$$\psi_1(x) = \left(\frac{1}{2a\sqrt{\pi}}\right)^{1/2} 2\left(\frac{x}{a}\right) e^{-\frac{x^2}{2a^2}} \text{ where } a = \sqrt{\frac{h}{2\pi m\omega}}$$

Is the wave function normalized? Explain your reasoning

5. (10 pts) A lightbulb that is operated at 60 W actually produces about 3 W of visible light. Most of the rest of the energy is heat. Assume the average wavelength of visible light is 550 nm. Evaluate how many visible photons such a lightbulb produce each second.

注意:背面有試題

6. For a simple harmonic potential well $V(x)=k(X-X_0)^2/2$,

    (a) (5 pts) When a particle with a mass of $m$ is in the well, what are the average values of the lowest kinetic and potential energies?

    (b) (5 pts) When a particle with a mass of $m$ is in the well, what is the root-mean-square speed at the first excited state?

    (c) (10 pts) If 20 fermions with a mass of $m$ are in the well, what is the mean energy of fermion at absolute temperature $T=0$ K? What is the Fermi energy of the system?

    (d) (5 pts) If 20 bosons with a mass of $m$ are in the well, what is the total energy of bosons at absolute temperature $T=0$ K?

7. (a) (5 pts) When a particle is in an infinite square-well potential ($V\to\infty$), how does the lowest energy of the particle change when the potential V becomes finite.

    (b) (10 pts) Continue (a), a particle is in a finite square-well potential as follows
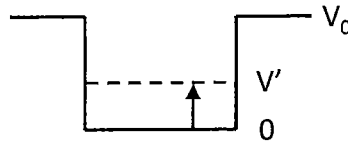


If the bottom of the square-well potential is increased from 0 to V', explain how the total energy and kinetic energy of the particle change?

8. (10 pts) Find the group and phase velocities of 10-MeV electrons.

注意:背面有試題