所別：　光電類

科目：　工程數學

本科考試可使用計算器，廠牌、功能不拘

計算題需計算過程，無計算過程者不予計分

1. a) (9 pt) Find the values of $\sqrt[3]{-1+\sqrt{3}i}$, where $i = \sqrt{-1}$

   b) (7 pt) Given that $\sqrt{16} = \sqrt{-(-16)} = i\sqrt{-16} = i\sqrt{16}i = i^2\sqrt{16} = -\sqrt{16}$

   There must be some mistakes in the above flow of arguments.
   Indicate where the mistake is and explain what the mistake is.

2. (16 pt) Evaluate the line integral

   $$\int_a^b \frac{y}{\sqrt{x^2+y^2}} ds$$
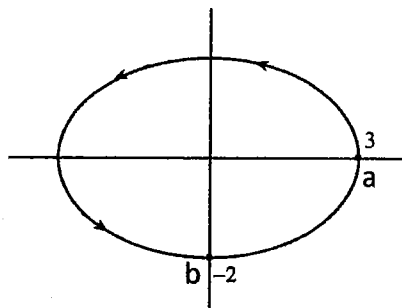

   Fig.1

   along the ellipse from "point a" to "point b" shown in Fig. 1, where the coordinate of "point a" is (3,0) and that of "point b" is (0,−2).

   You may use $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

3. Given that a periodic function f(x) = x in the interval $0 < x < \pi$
   $$= x + \pi \text{ for } -\pi < x < 0$$

   a) (1 pt) Graph the given function.
   b) (9 pt) Find the Fourier series of this function.
   c) (3 pt) What is the value of the Fourier series obtained above at $x = 0$ and $x = \pi$.
   d) (5 pt) Are there any other Fourier series that can accurately represent the above given function in the interval $0 < x < \pi$.　If the answer is NO, stop here. If the answer is YES, say something about these series such as what they represent outside the range $0 < x < \pi$.

注意：背面有試題

4. Solve the following initial value problems.

   (a) (10pt) $y'' + y' - 6y = 2\sin t$, $y(0) = 1$, $y'(0) = 0$.

   (b) (10pt) $y'' + 2y' + y = 3e^{-t}$, $y(0) = 0$, $y'(0) = 1$.

5. (10pt) Given that

$$h(x) = u(x+1) - u(x-1),$$

where $u(x-a) = \begin{cases} 1 & \text{if } x \geq a \\ 0 & \text{if } x < a \end{cases}$ is the Heaviside step function.

Find the Fourier transform of the following function

$$f(x) = h(x)\cosh x.$$

6. (10pt) Find the eigenvalues and eigenvectors of the matrix A, and calculate A^5.

$$A = \begin{pmatrix} 2 & 0 & 1+2i \\ 0 & 0 & 0 \\ 1-2i & 0 & -2 \end{pmatrix}$$

7. Calculate the curl of the following vector fields:

   (a) (5pt) $\mathbf{F} = \dfrac{1}{r}\left[(x-y)\hat{i} + (y-z)\hat{j} + (z-x)\hat{k}\right]$. Here $r = \sqrt{x^2 + y^2 + z^2}$.

   (b) (5pt) $\mathbf{G} = \mathbf{a} \times \nabla e^{\mathbf{b \cdot r}}$. Here $\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector, here $\mathbf{a}$, $\mathbf{b}$ are two constant vectors.

注意:背面有試題