國立中央大學101學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)　科目：近代物理　共 2 頁 第 1 頁
光電科學與工程學系碩士班 不分組(在職生)

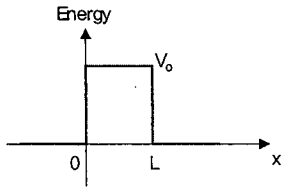
本科考試可使用計算器，廠牌、功能不拘　＊請在試卷答案卷（卡）內作答

1. A circle is at rest in S' with radius R_o. S' moves with velocity v with respect to the laboratory frame S, compute the area of the circle in S. (10%)

2. An electron with rest energy of 0.511 MeV moves with speed v=0.5c, find its kinetic energy and momentum (5%). If the electron moves along a circular path perpendicular to a uniform magnetic field of 1T, what is the radius of the circular path? (5%)

3. For a particle of mass m moving in the n=2 level in an one-dimensional infinite square well of width *L*, compute
   (a) the expectation value of $\langle x^2 \rangle$ (5%)
   (b) the energy expectation value at time *t* (5%)

4. A particle of mass *m* and energy *E* is incident on a potential barrier as shown in Figure below.



   (a) If $E<V_o$, what is the coefficient of transmission? (5%)
   (b) If a 20-eV electron is incident on a potential barrier of height 25 eV and width 0.1 nm, calculate the coefficient of transmission. (5%)

5. A particle of mass *m* is moving in an infinite one-dimensional parabolic potential well of the form, $V(x)=A(1-Bx)^2+C$,
   (a) What are the energy eigenvalue? (5%)
   (b) Find $\langle x \rangle$ and $\langle p \rangle$ for the ground-state wave function. (5%)

6. Please state and explain two theories/principles/phenomena that were discovered or postulated by Einstein in modern physics. What practical applications have been (or potential applications can be) realized based on these theories/principles/phenomena? (6%)

7. (a) Wave beats can be produced by the superposition of two waves with (slightly) different frequencies. Please sketch the beats wave superposed by two waves of same amplitude A_0 but different frequencies (ω and $\omega+\Delta\omega$) and wave numbers (k and $k+\Delta k$). Interpret the concept of the phase and group velocities of an optical wave in the medium and compare with that associated with a matter (de Broglie) wave. (6%)
   (b) From the wave beating phenomenon, please suggest how to produce a narrow wave packet (or wave group) in space. (3%)
   (c) Derive the Heisenberg uncertainty relation for position and momentum from the wave packet idea obtained in (b). (5%)

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(d) Use the uncertainty relation you obtained in (c) to explain why an electron can't be observed (with the help of light of wavelength λ) without changing its momentum. (4%)

8. (a) Electrons can be used to generate photons, and vice versa. Take some physical phenomena observable in nature or in experiments as examples to show and explain the two energy conversion processes (i.e., electrons to photons and photons to electrons). Please give at least two examples for each process. (6%)
(b) Without the use of de Broglie's hypothesis, please derive the condition for electron orbit stability: $r_n = n\lambda/2\pi$ (where n is an integer and λ is the wavelength), based on Bohr's model postulating that the angular momentum of an orbital electron must be an integral multiple of $\hbar$ ($\equiv h/2\pi$; the reduced Planck's constant). Show the energy levels of the hydrogen can be expressed as $E_n = chR(\frac{1}{n^2})$, where c is the light speed in vacuum and R is a constant called Rydberg constant. (7%)
(c) From the x-ray spectrum of a target bombarded by fast electrons, one can explore the energy levels of the target and know the element constituent in the target. Show the atomic number Z of the target element can be deduced from the K_α photon frequency measured in the x-ray spectrum via the relationship $\nu(K_\alpha) = 3cR(Z-1)^2/4$. (5%)

9. (a) Find the ground-state configurations of the following transition elements by using the information shown in Fig. A diagram: $_{24}$Cr, $_{30}$Zn, $_{60}$Nd, and $_{64}$Gd. (5%)
(b) Why the chemical properties of transition elements in the same Period of the periodic table can be similar? (3%)

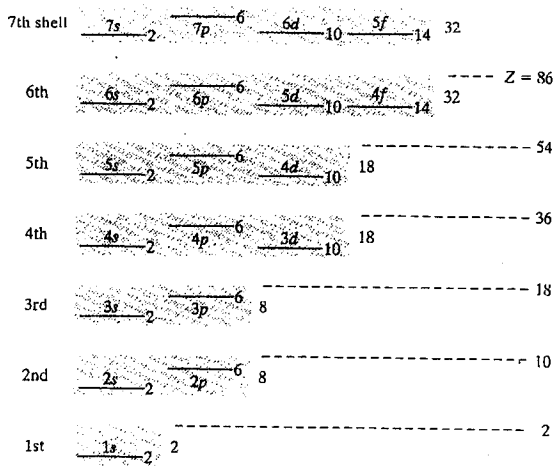

Fig. A. Schematic diagram showing the order in which levels are occupied as one considers atoms with successively higher atomic number Z.

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