

國立中央大學100學年度碩士班考試入學試題卷

所別：光電科學與工程學系碩士班 不分組(一般生)

科目：工程數學

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本科考試可使用計算器，廠牌、功能不拘

*請在試卷答案卷(卡)內作答

參考用

1. (15%) Find the particular solutions of the following equations:
 - (a) (5%) $x'' + 6x' + 13x = 13t + 19$.
 - (b) (5%) $x'' + 6x' + 13x = e^{-3t}$.
 - (c) (5%) $x'' + 6x' + 13x = e^{-3t} \cos(2t)$.

2. (15%) Consider the matrix $A = \begin{bmatrix} 4 & 6 \\ 3 & 1 \end{bmatrix}$.
 - (a) (3%) Find the inverse of A.
 - (b) (5%) Find the eigenvalues and the corresponding eigenvectors of A.
 - (c) (4%) Solve the linear system $A\bar{x} = \bar{b}$ for $\bar{b}_1 = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$ and $\bar{b}_2 = \begin{bmatrix} 13.9 \\ 7.1 \end{bmatrix}$.
 - (d) (3%) Compute $\frac{|\bar{x}_1 - \bar{x}_2|}{|\bar{b}_1 - \bar{b}_2|}$ from your solutions in part (c). As $\bar{b}_1 - \bar{b}_2 = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$ can be viewed as a measurement noise in \bar{b} , how does this particular noise vector propagate into the solution?

3. (15%) Find $\nabla \times [rf(r)]$, where $r = xi + yj + zk$ and $f(r)$ is differentiable.

4. (15%) The growing rate of your vegetable in the Happy Farm follows the differential equation $g(x) = \frac{dx}{dt} = -x^2 + 10x$ where x is the quantity of your vegetable.
 - (a) (5%) Find the solution of x . Sketch some solutions.
 - (b) (5%) What is the largest growing rate $g(x)$ that will allow for a constant quantity of your vegetable?
 - (c) (5%) You can maintain the growing rate $g(x)=3$ over a long period of time, with a constant vegetable mass. What must that mass be, approximately?

5. (15%) Consider a parabola $y = ax^2 + bx + c$ passing through the points $(x_1, y_1) = (-2, 1.4)$, $(x_2, y_2) = (0, -0.6)$, and $(x_3, y_3) = (3, 0.9)$. The curve-fitting problem can be modeled as a linear system:

$$A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Construct the system matrix A and solve for the parameters $[a \ b \ c]^T$. Plot the parabola and find the (x, y) coordinate of the minimum point.

注意：背面有試題

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6. (25%) The Fourier transform pair is given by

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{i\omega t} d\omega$$

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt$$

(a) (5%) Explain, in as much detail as you can without resorting to any physical examples, the meaning of the expression for $x(t)$.

(b) (10%) Calculate, showing all the work, the Fourier transform of $x(t)$, where

$$x(t) = \frac{30}{25\omega^2 + 9}$$

(c) (10%) Given a function $f(t) = t^2 [\delta(t + \frac{1}{2}) - \delta(t - \frac{1}{2}) + \delta(t)]$, determine the Fourier transform of $g(t) = \int_{-\infty}^t f(\tau) d\tau$. Express your answer in simplest form.

參考用