國立中央大學99學年度碩士班考試入學試題卷

所別：統計研究所碩士班 不分組(一般生)　科目：數理統計　共 2 頁 第 1 頁

不分組(在職生)

＊請在試卷答案卷（卡）內作答

＊本科考試可使用計算器，廠牌、功能不拘

1. Let $Y_1, Y_2, \ldots, Y_n$ be i.i.d. Poisson random variables with means λ. Find the conditional distribution of $\sum_{i=1}^{m} Y_i$ given $\sum_{i=1}^{n} Y_i = k$, where $m < n$. Identify the distribution. [10 %]

2. Roll a fair die twice.

   (a) Write down the sample space and find the probability that the resulting sum is even. [10 %]

   (b) If the same die is rolled 50 times, find the approximate probability, to four decimal digits, that 6 appears at least 8 times. [10 %]

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution with mean λ. Find the MLE (maximum likelihood estimator) and its asymptotic distribution of $P(X_1 = 0)$ for large n. [15 %]

4. Let X_1 and X_2 be i.i.d. random variables with probability mass function $p(x|\theta)$, where θ could be either θ_0 or θ_1, given by

	x	
	1	2
$p(x\|\theta_0)$	0.5	0.5
$p(x\|\theta_1)$	0.3	0.7

   Find the MLE of θ based on observations X_1 and X_2. [10 %]

5. The entries in the following table represent the joint probability mass function of the random vector (X, Y).

		Y		
		1	0	-1
X	1	1/12	0	1/12
	0	0	1/3	1/3
	-1	0	1/6	0

   Find $Cov(X, Y)$. Are X and Y independent? [10 %]

6. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from the probability density function given by

$$f(y|\theta) = \frac{1}{\theta}\, m\, y^{m-1}\, e^{-y^m/\theta}, y > 0, \theta > 0$$

   and zero, otherwise, with m denoting a known constant.

   (a) Find the uniformly most powerful test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. [10 %]

   (b) If the test in (a) is to have $\theta_0 = 100$, α (type I error probability) $=.05$, and β (type II error probability) $=.05$ when $\theta = 400$, find the appropriate sample size and critical region. [10 %]

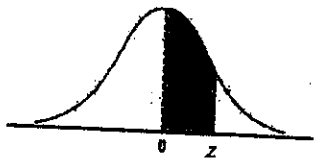
7. Suppose that $X_1, X_2, \ldots, X_{n_1}$, $Y_1, Y_2, \ldots, Y_{n_2}$, and $W_1, W_2, \ldots, W_{n_3}$ are independent random samples from normal distributions with respective unknown means μ_1, μ_2 and μ_3 and variances σ_1^2, σ_2^2 and σ_3^2, but it is known that σ_1^2, σ_2^2 and σ_3^2 are all equal. Suppose that we want to estimate $\theta = a_1\mu_1 + a_2\mu_2 + a_3\mu_3$, with a_1, a_2 and a_3 being known constants. Give a confidence interval for θ with confidence coefficient $1 - \alpha$. [15 %]

參考用

注意：背面有試題

所別：統計研究所碩士班 不分組(一般生)
不分組(在職生)　科目：數理統計　共 2 頁 第 2 頁

＊請在試卷答案卷（卡）內作答
＊本科考試可使用計算器，廠牌、功能不拘



標準常態累加機率值表：P(0<Z<z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

參考用

注意：背面有試題