國立中央大學九十一學年度碩士班研究生入學試題卷

所別：統計研究所 不分組 科目：數理統計 共一頁 第一頁

請依題目編號逐一作答

A. 簡答題(只需給答案即可!) 60%

(A1). Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample from an exponential distribution that has mean of 1, then $\Pr(Y_1 > 1)$=? (6%)

(A2). Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample from the uniform distribution $U(0, \theta)$.

(a) Find the maximum likelihood estimator of θ. (6%)

(b) Find the sufficient statistics of θ. (6%)

(c) Find a constant c so that cY_n is an unbiased estimator of θ. (6%)

(A3). The length in centimeters of $n = 8$ fish were, 5.0, 3.9, 5.2, 2.8,6.1, 6.4, 2.7, 2.3. Let m_0 denote the median of length. To test the hypothesis $H_0 : m_0 = 3.7$ against $H_1 : m_0 > 3.7$. Find value of the Wilcoxon sign rank-sum statistics. (6%)

(A4). Let $X_1, X_2, \cdots, X_n$ be a random sample from Bernoulli distribution with parameter p. Suppose that p has a $U(0, 1)$ prior density, find the Bayes estimator for p. (6%)

(A5). Let $X_1, X_2, \cdots, X_{100}$ be a random sample from a Poisson distribution with a mean of 1. Approximate $\Pr(75 < \Sigma_{i=1}^{100} X_i \leq 82)$, by using Central Limit Theorem. (6%)

(A6). Let $X_1, X_2, \cdots, X_n$ be a random sample from normal distribution with mean μ, where $\mu \geq 0$ and variance 1. Find the maximum likelihood estimator for μ. (6%)

(A7). If X and Y are independent random variables, what is the regression equation by regressing Y on X (6%)

(A8). Thirty samples of data for the heights (x) and weights (y) were obtained. After algebras, the results gave $\Sigma x_i = 60$, $\Sigma y_i = 90$, $\Sigma x_i^2 = 300$, $\Sigma y_i^2 = 750$, $\Sigma x_i y_i = 420$. Find the equation of the least squares line of y on x. (6%)

B. 計算及証明題(請列出計算及証明過程) 40%

(B1). Let $X_1, X_2, \cdots, X_n$ be a random sample from Normal $(\mu, 1)$. Given $\mathbf{H}_0$: μ=0 and $\mathbf{H}_1$: μ=1, how large should n be chosen to guarantee α (Type I error) and β (Type II error) = 0.05. (10%)

(B2). If the random variable X is $N(0, 1)$. Show that the random variable $V = X^2$ is $\chi^2(1)$. (10%)

(B3). Let $X_1, X_2, \cdots, X_n$ be a random sample from the normal distribution, $N(\mu, \sigma^2)$. Find a $100(1-\alpha)\%$ confidence interval for σ^2. (10%)

(B4). Let $X_1, X_2, \cdots, X_n$ be a random sample from the normal distribution, $N(\mu, \sigma^2)$, where σ^2 is unknown. Calculate the expected length of a 95% confidence interval for μ. (10%)