國立中央大學八十六學年度碩士班研究生入學試題卷

所別：統計研究所 組 科目：數理統計 共 1 頁 第 1 頁

Please answer the following questions in order!

I. Suppose $X_1, X_2, \cdots, X_n$ be a random sample from Normal(μ, σ^2), while μ and σ^2 are unknown. Define two estimators of σ^2 as follows:

$$S_1^2 = \sum_{i=1}^{n}(X_i - \bar{X}_n)^2/n \quad \text{and} \quad S_2^2 = \sum_{i=1}^{n}(X_i - \bar{X}_n)^2/(n-1),$$

here $\bar{X}_n$ is the sample mean. Compare the mean squared errors between S_1^2 and S_2^2. (15%)

II. Suppose that the proportion p of defective items in a large population of items is unknown, and that it is desired to test H_0: $p = 0.4$ against H_1: $p < 0.4$. If a random sample of 100 item is drawn from the population. Let Y denote the number of defective items in the sample, and consider a test procedure such that the critical region contains all the outcomes for which $Y \le 32$. Determine the size of the test (8%) and the power at $p = 0.2$ (4%), by using central limit theorem.

III. For any random variables X and Y, show that the correlation coefficient between X and Y, denote by ρ_{XY}, satisfies $-1 \le \rho_{XY} \le 1$. (10%)

IV. Let $X_1, X_2, \cdots, X_n$ be a random sample from Normal(μ, σ^2), where μ and σ^2 are unknown.

(1). Find a $(1-\alpha)100\%$ confidence interval of μ. (5%)

(2). Convert the confidence interval obtained in (a) into a level 2α testing procedure for H_0: $\mu = \mu_0$ against H_1: $\mu \ne \mu_0$. (5%)

V. Just give answers to the following problems. Be careful of calculations!

(1). Suppose that a point (X, Y) is chosen at random from the cirsle S, where S=$\{(x,y) : (x+3)^2 + (y-1)^2 \le 13\}$. Then Prob$(Y \ge 1 | X = 0)$= ? (5%)

(2). Let X be a discrete random variable with cdf $F_X(x)$ and Y=$F_X(X)$. Which one of the following statements is correct? (5%)

(a). Y follows uniform(0,1).

(b). Y is stochastically larger than uniform(0,1).

(c). Y is stochastically smaller than uniform(0,1).

(3). In problem **I**, which one, S_1^2 or S_2^2, is unbiased ? (5%)

(4). Let the probability p_n that a family has exactly n children be αp^n when $n \ge 1$, and $p_0 = 1 - \alpha p(1 + p + p^2 + \cdots)$. Suppose that all sex distributions of n children have the same probability. Find the probability that a family has exactly k boys, $k \ge 1$. (5%)

(5). For any random variables X and Y,

(a). is X and $Y - E(Y|X)$ correlated or not ? (5%)

(b). let $\min_{g(X)} E[(Y - g(X))^2] = E[(Y - g^*(X))^2]$, then $g^*(X)$= ? (5%)

(6). Let $\min_a E|X - a| = E|X - a(X)|$, then $a(X)$= ? (5%)

VI. Give definitions to following statements:

(1). Chebychev Inequality (3%)

(2). Weak Law of Large Numbers (3%)

(3). Central Limit Theorem (3%)

(4). Cramér-Rao Inequality (3%)

(5). Neyman-Pearson Lemma (3%)

(6). Lehmann-Scheffé Theorem (3%)

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