

# 國立中央大學八十六學年度碩士班研究生入學試題卷

所別: 統計研究所 組 科目: 數理統計 共 1 頁 第 1 頁

Please answer the following questions in order!

I. Suppose  $X_1, X_2, \dots, X_n$  be a random sample from  $\text{Normal}(\mu, \sigma^2)$ , while  $\mu$  and  $\sigma^2$  are unknown. Define two estimators of  $\sigma^2$  as follows:

$$S_1^2 = \sum_{i=1}^n (X_i - \bar{X}_n)^2 / n \quad \text{and} \quad S_2^2 = \sum_{i=1}^n (X_i - \bar{X}_n)^2 / (n-1),$$

here  $\bar{X}_n$  is the sample mean. Compare the mean squared errors between  $S_1^2$  and  $S_2^2$ . (15%)

II. Suppose that the proportion  $p$  of defective items in a large population of items is unknown, and that it is desired to test  $H_0: p = 0.4$  against  $H_1: p < 0.4$ . If a random sample of 100 item is drawn from the population. Let  $Y$  denote the number of defective items in the sample, and consider a test procedure such that the critical region contains all the outcomes for which  $Y \leq 32$ . Determine the size of the test (8%) and the power at  $p = 0.2$  (4%), by using central limit theorem.

III. For any random variables  $X$  and  $Y$ , show that the correlation coefficient between  $X$  and  $Y$ , denote by  $\rho_{XY}$ , satisfies  $-1 \leq \rho_{XY} \leq 1$ . (10%)

IV. Let  $X_1, X_2, \dots, X_n$  be a random sample from  $\text{Normal}(\mu, \sigma^2)$ , where  $\mu$  and  $\sigma^2$  are unknown.

(1). Find a  $(1 - \alpha)100\%$  confidence interval of  $\mu$ . (5%)

(2). Convert the confidence interval obtained in (a) into a level  $2\alpha$  testing procedure for  $H_0: \mu = \mu_0$  against  $H_1: \mu \neq \mu_0$ . (5%)

V. Just give answers to the following problems. Be careful of calculations!

(1). Suppose that a point  $(X, Y)$  is chosen at random from the circle  $S$ , where  $S = \{(x, y) : (x+3)^2 + (y-1)^2 \leq 13\}$ . Then  $\text{Prob}(Y \geq 1 | X = 0) = ?$  (5%)

(2). Let  $X$  be a discrete random variable with cdf  $F_X(x)$  and  $Y = F_X(X)$ . Which one of the following statements is correct? (5%)

(a).  $Y$  follows uniform(0,1).

(b).  $Y$  is stochastically larger than uniform(0,1).

(c).  $Y$  is stochastically smaller than uniform(0,1).

(3). In problem I, which one,  $S_1^2$  or  $S_2^2$ , is unbiased? (5%)

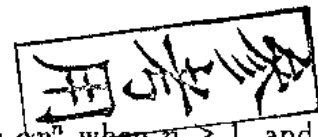
(4). Let the probability  $p_n$  that a family has exactly  $n$  children be  $\alpha p^n$  when  $n \geq 1$ , and  $p_0 = 1 - \alpha p(1 + p + p^2 + \dots)$ . Suppose that all sex distributions of  $n$  children have the same probability. Find the probability that a family has exactly  $k$  boys,  $k \geq 1$ . (5%)

(5). For any random variables  $X$  and  $Y$ ,

(a). is  $X$  and  $Y - E(Y|X)$  correlated or not? (5%)

(b). let  $\min_{g(X)} E[(Y - g(X))^2] = E[(Y - g^*(X))^2]$ , then  $g^*(X) = ?$  (5%)

(6). Let  $\min_a E|X - a| = E|X - a(X)|$ , then  $a(X) = ?$  (5%)



VI. Give definitions to following statements:

- |                            |      |                                |      |
|----------------------------|------|--------------------------------|------|
| (1). Chebychev Inequality  | (3%) | (2). Weak Law of Large Numbers | (3%) |
| (3). Central Limit Theorem | (3%) | (4). Cramér-Rao Inequality     | (3%) |
| (5). Neyman-Pearson Lemma  | (3%) | (6). Lehmann-Scheffé Theorem   | (3%) |