國立中央大學八十五學年度碩士班研究生入學試題卷

所別：統計研究所　甲組　　科目：機率論　　共 1 頁　第 1 頁

I、Let A, B, C be three measurable events in a probability space. Suppose that A and B are independent and $P(A)=0.6$, $P(B)=0.7$, $P(C|B)=0.2$ and $P(C|AB)=0.1$. Let A^c be the complement of A. Find

(1) $P(A|B)$ (5%)

(2) $P(BC|A)$ (5%)

(3) $P(C|A^cB)$ (5%)

(4) $P(A^c|BC)$ (5%)

II、(1) Show that $[Cov(X,Y)]^2 \le Var(X)\, Var(Y)$ (5%)

(2) Show that for arbitrary measurable events $A_1, A_2, ..., A_n$ in a probability space, the following inequality holds:

$$P(A_1A_2...A_n) \ge 1 - \sum_{i=1}^{n} P(A_i^c).$$ (5%)

III、(1) Let $g(x) > 0$ for $x>0$ be a monotonically increasing function. Suppose $E[g(|X|)] = M$ exists. Show that $P(|X| \ge x) \le M/g(x)$ for all $x>0$. (5%)

(2) Use (1) to prove the Chebyshev inequality. (5%)

(3) Suppose that S_n follows a binomial distribution $b(n,p)$ for $n=1, 2, ...$ Show that S_n/n converges to p in probability as $n \to \infty$ by using the Chebyshev inequality. (5%)

IV. If f and F be the pdf and cdf of an absolutely continuous distribution. For $n \ge 2$, define

$$f_n(y_1, y_2, ..., y_n) = f(y_n) \prod_{i=1}^{n-1} f(y_i)/[1-F(y_i)], \quad y_1 \le y_2 \le ... \le y_n$$
$$= 0, \text{ otherwise.}$$

(1) Suppose

$$f(y) = e^{-y}, \ y \ge 0$$
$$= 0, \text{ otherwise}$$

and $Y_1, Y_2, ..., Y_n$ are the n random variables with f_n as their joint pdf. Define $W_i = Y_i - Y_{i-1}$, $i=1, 2, ..., n$, with $Y_0 = 0$. Determine whether $W_1, W_2, ..., W_n$ are independant. (10%)

(2) Determine the distribution of Y_n using the information in (1). (5%)

(3) Show that Y_n is asymptotic normal and determine the norming constants. (5%)

V. Let (X,Y) have the following joint pdf

$$f(x,y) = a \quad \text{for } 0 \le y \le x \le 1 \text{ or } 0 \le x \le 1/2,\ x+1/2 \le y \le 1$$
$$= 0 \quad \text{otherwise.}$$

Find

(1) a (5%)

(2) Marginal pdf of Y (5%)

(3) $E[X|Y=y]$ (5%)

(4) $P(2X+Y \le 3/2)$ (5%)

VI. Let X have a pdf f(x) which is a mixture of two pdfs f_1 and f_2 as follows:

$$f(x) = (1-\varepsilon)\, f_1(x) + \varepsilon\, f_2(x), \quad 0<\varepsilon<1.$$

Suppose now $f_i(x)$ is the pdf of a normal distribution with mean μ_i and variance σ_i^2, $i=1, 2$. Find

(1) $E[X]$ (5%)

(2) $Var(X)$ (5%)

(3) The moment generating function of X. (5%)

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