1. State without proof a version of the Fundamental Theorem of Calculus.
(8%)

2. (a) What is the Integral Test ? (6%)

   (b) Determine if the series $\sum_{n=1}^{\infty} \frac{\log n}{n}$ is convergent.
   (6%)

3. Maximize $x^2 + xy - 3y^2$ subject to the constraint $2 - x - 2y = 0$. (10%)

4. Give the series expansions of the following functions and indicate the intervals of convergence. (a) e^x. (b) $\sin x$, (c) $\log(1 - x)$. (10%)

5. Describe the Newton-Raphson algorithm. (10%)

6. (a) Let A be a $m \times n$ matrix and B be a $n \times m$ matrix. Show that $Tr(AB) = Tr(BA)$, where $Tr(\cdot)$ denotes the trace of a square matrix.
   (6%)

   (b) Show that there are <u>no</u> $n \times n$ matrices A and B such that $AB - BA = I$, where I denotes the $n \times n$ identity matrix. (4%)

7. Let $A : R^2 \to R^3$ be defined by

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ x - y \\ 2x + y \end{bmatrix}.$$

Find

   (a) the Kernel of A, (4%)

   (b) an orthonormal basis for the range of A.
   (6%)

8. Let A and B be similar matrices (i.e. $A = P^{-1}BP$ for some non-singular P). Show that (10%)

   (a) A' and B' are similar, where A' denotes the transpose of matrix A.

   (b) rank A = rank B,

   (c) A is nonsingular if and only if B is nonsingular,

   (d) $Tr(A) = Tr(B)$,

   (e) If A and B are nonsingular, then A^{-1} and B^{-1} are similar.

9. Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A. (4%)

   (b) Find the eigenvalues of A and the corresponding eigenvectors. (6%)

10. Let $C(-\infty, \infty)$ denote the set of all real-valued continuous functions, i.e. $C(-\infty, \infty) = \{f : f \in C \text{ on } (-\infty, \infty)\}$. Which of the following <u>subsets</u> are subspaces of the vector space $C(-\infty, \infty)$? (10%)

    (1) All nonnegative functions.

    (2) All constant functions.

    (3) All functions f such that $f(0) = 0$.

    (4) All functions f such that $f(0) = 5$.

    (5) All differentiable functions.

    (6) All integrable functions.

    (7) All bounded functions.

    (8) All functions that are integrable on $[a, b]$.

    (9) All functions that are bounded on $(-\infty, \infty)$.

    (10) All polynomial functions.