所別：**數學系碩士班　不分組** 科目：**離散數學**

以下題目，任選五題作答，每題二十分，滿分一百。若答題超過五題，第六題起不予計分。

1. If C is a cycle, and e is an edge connecting two nonadjacent vertices of C, then we call e a *chord* of C. Prove that (a) if every vertex of a graph G has degree at least **3**, then G contains a cycle with a chord; (b) if G has $n \geq 4$ vertices and $2n-3$ edges, then G contains a cycle with a chord.

2. Show that a number with 160 digits has a prime power divisor that is at least 100.

3. We are given $n+1$ numbers from the set $\{1, 2, \cdots, 2n\}$. Prove that there are two numbers among them such that one divides the other.

4. Let $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that (a) $F_{a+b+1} = F_{a+1}F_{b+1} + F_a F_b$ for any positive integers a and b; (b) if n is a multiple of k, then F_n is a multiple of F_k; (c) for every positive integer m there is a positive integer n such that F_n is a multiple of m.

5. Determine the number of integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ which satisfy $1 \leq x_1 \leq 5$, $-2 \leq x_2 \leq 4$, $0 \leq x_3 \leq 5$, $3 \leq x_4 \leq 9$.

6. G is a planar graph. Show that G contains a vertex with degree ≤ 5.

7. Prove that a simple graph of order $n \geq 3$, in which the sum of the degrees of each pair of nonadjacent vertices is at least n, has a Hamiltonian cycle.