國立中央大學九十一學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 機率與統計 共 1 頁 第 1 頁

1. Let X be a random variable which is nonnegative and bounded above by a constant c, $c>0$. (10分)
   (a) Show that $P(X \geq k) \leq \frac{c}{k}$ for any $k>0$.
   (b) Show that $Var(X) \leq \frac{c^2}{4}$.

2. Let X have density $f(x)=\frac{1}{\theta}\exp\{-\frac{x}{\theta}\}$, $x \geq 0$, $\theta>0$. Let $\bar{X}=\frac{X_1+X_2}{2}$, where X_1, X_2 are i.i.d. with density $f(x)$. (10分) Find $P(\bar{X} \geq t)$.

3. Let (X,Y) have joint density function $f(x,y)=8xy$, $0<x<y<1$ and zero elsewhere.
   (a) Are X and Y independent? (20分)
   (b) Find $P\{\frac{1}{4}<X<\frac{1}{2} \mid Y=\frac{3}{8}\}$
   (c) Find $P\{\frac{1}{4}<X<\frac{1}{2} \mid \frac{1}{4}<Y<\frac{3}{8}\}$.

4. Let X_1, X_2, X_3 be random samples with common density function $f(x)=1$, $0<x<1$ and zero elsewhere. (20分)
   (a) Find $P(|X_2-X_3|>t)$
   (b) Find $E|X_2-X_3|$
   (c) Find $P(|X_{(2)}-X_{(3)}|<\frac{1}{4})$

5. Let $X_1, \cdots, X_n$ denote a random sample from the density given by $f(x|\alpha,\theta)=\frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}$, $x>0$, $\theta>0$ where $\alpha>0$ is known. (20分)
   (a) Find the maximum-likelihood estimator $\hat{\theta}$ of θ.
   (b) Find the expected value and variance of $\hat{\theta}$.

6. Let X be a random sample from the density given by $f(x|\theta)=\theta x^{\theta-1}$, $0<x<1$, and zero elsewhere. (20分)
   (a) Find the most powerful test with significance level $\alpha=0.1$ to test $H_0: \theta=1$ against $H_a: \theta=2$
   (b) Find the type II error of this test.

參考用