

國立中央大學九十學年度碩士班研究生入學試題卷

所別: 數學系 不分組 科目: 微分方程 共 一 頁 第 一 頁

1. (30%)

(1) Find the general solution of the differential equation

$$\frac{y^2}{2} + 2ye^t + (y + e^t)\frac{dy}{dt} = 0.$$

(2) Solve $t^2y' + 2ty - y^3 = 0$, $t > 0$.

2. (15%) Solve the following initial value problem

$$\frac{dX}{dt} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} X + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t}, \quad X(0) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

3. (15%) Let y_1 and y_2 be solutions of the equation:

$$t^2y'' + ty' + (t^2 - n^2)y = 0.$$

on the interval $0 < t < \infty$, with $y_1(1) = 1$, $y_1'(1) = 0$, $y_2(1) = 0$, and $y_2'(1) = 1$. Compute wronskian $W[y_1, y_2](t)$.

4. (10%) Suppose that $T > 0$ and f is a continuous and periodic function on $[0, \infty)$ with period T : $f(t + T) = f(t)$ for all $t > 0$. Show that for $s > 0$, the Laplace transformation of f is

$$\mathcal{L}[f](s) = \frac{\int_0^T e^{-st} f(t) dt}{1 - e^{-sT}}.$$

5. (15%) Find a series solution in powers of x of the equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

6. (15%) Find all equilibrium solutions of the system

$$\begin{aligned} \frac{dx}{dt} &= 1 - xy, \\ \frac{dy}{dt} &= x - y^3. \end{aligned}$$

and determine whether they are stable or unstable.