Note: $\mathbb{R}$ denotes the real line.
$C^2(\mathbb{R}^3)$ denotes the set of twice continuously differentiable functions on $\mathbb{R}^3$.

1. (10%) Let $f(x,y) = (x+3y, 4x-y, 2x+y)$ and $g(u,v,w) : \mathbb{R}^3 \to \mathbb{R}$ with $g \in C^2(\mathbb{R}^3)$. Let $h = g \circ f$. Find the partial derivative $\dfrac{\partial^2 h}{\partial y \partial x}$.

2. (15%) Find $\iiint_D z\,dV$, where D is the region bounded by the surface $36x^2 + 9y^2 + 4zz^2 = 36$ and the surface $36x^2 + 9y^2 - 4z^2 = 0$ with $z > 0$.

3. (15%) Give a function $f(x)$ on an unbounded interval I such that $f(x)$ is continuous on I but not uniformly continuous on I. Prove your assertions.

4. (20%) Let $f : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \dfrac{1}{n} & \text{if } x \text{ is rational, } x = \dfrac{m}{n}, \text{ where } n > 0 \text{ and } m, n \text{ are relatively prime.} \\ 0 & \text{if } x = 0 \text{ or if } x \text{ is irrational.} \end{cases}$$

Prove that $f(x)$ is Riemann integrable over $[0,1]$ directly from the definition of Riemann integrals, and evaluate $\int_0^1 f(x)\,dx$.

5. (20%) Let $f : \mathbb{R} \to \mathbb{R}$ be differentialbe on $\mathbb{R}$. If $0 \le f'(x) \le f(x)$ for all $x \in \mathbb{R}$, and if there is an $x_0 \in \mathbb{R}$ such that $f(x_0) = 0$, then prove that $f(x) = 0$ for all $x \in \mathbb{R}$.

6. (20%) For each n, let $f_n(x)$ be an increasing function on $[a,b]$. If $\lim_{n\to\infty} f_n(x) = f(x)$, $x \in [a,b]$, and $f(x)$ is continuous on $[a,b]$, then prove that $f_n \to f$ uniformly on $[a,b]$.