國立中央大學八十九學年度碩士班研究生入學試題卷

1 (10 points) A natural cubic spline S on [0,2] is defined by

$$f(x) = \begin{cases} 1 + 2x - 3x^2, & x \in [0,1) \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3, & x \in [1,2] \end{cases}$$

Find a, b, c and d

2 (10 points) Find a polynomials (Lagrage or Newton interpolating polynomials) of least degree that interpolate the set of data

x	2	0	3
$f(x)$	11	7	28

3 (10 points) Find a formula

$$\int_{-\pi}^{\pi} f(x)\cos x dx \approx A_0 f(-\frac{3}{4}\pi) + A_1 f(-\frac{1}{4}\pi) + A_2 f(\frac{1}{4}\pi) + A_3 f(\frac{3}{4}\pi)$$

that is exact for the polynomials of degree three.

4 (10 points) Starting with (0, 1), carry out an iteration of Newton's method on system:

$$f(x) = \begin{cases} 4x_1^2 - x_2^2 = 0 \\ 4x_1 x_2^2 - x_1 = 0 \end{cases}$$

5 A sequence $\{p_n\}$ is said to be **superlinearly convergent** to p if a sequence $\{c_n\}$ converging to zero exists with

$$|p_{n+1} - p_n| \le c_n |p_n - p|$$

(a) (10 points) Show that if $\{p_n\}$ is superlinearly convergent to p, then $\{p_n\}$ is linearly convergent to p.

(b) (10 points) Show that $p_n = \frac{1}{n^n}$ is superlinearly convergent to zero but is not quadratically convergent to zero.

6 (10 points) What is the condition number for the evaluation of $f(x) = \sin^{-1} x$? Where is it large?

7 (a) (10 points) Prove that the normal equations

$$A^* A x = A^* b$$

has a unique solution x, and this x solve the least-square problem: $\min ||Ax - b||_2$.

(b) (5 points) Prove that if A is an $m \times n$ matrix of rank n, then $A * A$ is Hermitian and positive definite. The Cholesky factorization may therefore be used to solve the normal equations.

8 (a) (5 points) State the Gershgorin's Theorem.

(b) (10 points) Use Gershorgin's Theorem to prove that a diagonally dominant matrix does not have zero as an eigenvalue, and is therefore nonsingular.