

國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 離散數學 共 2 頁 第 1 頁

1. (20%) $n+1$ distinct integers are chosen from $1, 2, 3, \dots, 2n$.

Show that ① among the integers chosen there are two such that one of them is divisible by the other.

② among the integers chosen there are two which are relatively prime.

2. (10%) z is a root of the equation $x^k - a_1 x^{k-1} - a_2 x^{k-2} - \dots - a_k = 0$

Show that $\{z^n\}_{n=0}^{\infty}$ is a solution of

$$h_n = a_1 h_{n-1} + a_2 h_{n-2} + \dots + a_k h_{n-k}, \quad n = k, k+1, \dots$$

3. (20%) Consider the recurrence relation:

$$(I) f_n = f_{n-1} + f_{n-2} \quad \text{for } n = 2, 3, 4, \dots$$

$$\text{with } f_0 = f_1 = 1.$$

① Solve the above relation (I) so as to express f_n in the form $C_1 z_1^n + C_2 z_2^n$.

② Show that $f_n = \binom{n}{0} + \binom{n-1}{1} + \binom{n-2}{2} + \dots + \binom{1}{n-1} + \binom{0}{n}$ is a solution of (I).

4. (20%) Show that every $(lm+1)$ -term sequence of real numbers contains either an $(l+1)$ -term increasing subsequence or an $(m+1)$ -term decreasing subsequence.

參考用

注意: 背面有試題

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5. (10%) ① State "System of Distinct Representative Theorem" (S.D.R. Theorem) (or Marriage Theorem, or Matching Theorem)
- (20%) ② Suppose $r < n$. Show that every r -by- n Latin rectangle can be extended to a $(r+1)$ -by- n Latin rectangle.

參考用