

國立中央大學八十八學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 微分方程 共 1 頁 第 1 頁

1. (20%) Solve the following differential equations respectively.

(a) $(3xy + y^2) + (x^2 + xy)y' = 0, y(1) = 1$

(b) $y'' - 2y' - 3y = 2e^x - 10 \sin x, y(0) = 2, y'(0) = 4.$

2. (20%) Solve the following initial value problem. Describe the behavior of the solution $x(t)$ as $t \rightarrow \infty$.

$$x' = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} x, \quad x(0) = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}.$$

3. (20%) Consider the Chebyshev differential equation

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0, \quad \alpha \text{ is a constant.}$$

(a) Determine two linearly independent solutions in powers of x for $|x| < 1$.

(b) Show that if α is a nonnegative integer n , then there is a polynomial solution of degree n .

(c) Find a polynomial solution for each of the cases $\alpha = n = 0, 1, 2$, and 3 .

4. (20%) Consider the following system of differential equations

$$\begin{cases} \frac{dx}{dt} = x - y - x(x^2 + y^2) \\ \frac{dy}{dt} = y + x - y(x^2 + y^2) \\ (x(0), y(0)) \neq (0, 0). \end{cases}$$

Use polar coordinate (r, θ) , $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1} \frac{y}{x}$ to derive $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ and discuss the behavior of the solution $(x(t), y(t))$ as $t \rightarrow \infty$.

5. (20%) Let $A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ and let $x(t)$ be a solution of $x' = Ax$, not identically zero. Show that the curve $x(t)$ is one of the following form :

(a) a circle if $a = 0$.

(b) a spiral inward toward $(0,0)$ if $a < 0, b \neq 0$.

(c) a spiral outward away $(0,0)$ if $a > 0, b \neq 0$.

(d) What effect has the sign of b on the spiral in (b) and (c) ?

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