

# 國立中央大學八十七學年度碩士班研究生入學試題卷

所別: 數學研究所 不分組 科目: 複變函數論 共 1 頁 第 1 頁

- 20% 1. Find a conformal mapping  $f$  of the open half-disc  $\{z: |z| < 1, \operatorname{Im} z > 0\}$  onto the full disk  $\{z: |z| < 1\}$ .
- 10% 2. Let  $f(z) = u + iv$  be an entire function with the property that  $u(z) \leq 0$  for all  $z$ . Prove that  $f$  is a constant function.
- 10% 3. If  $f(z)$  is analytic and  $|f(z)| \leq M$  for  $|z| \leq R$ , find an upper bound for  $|f^{(n)}(z)|$  in  $|z| \leq \rho < R$ .
- 10% 4. Find the Laurent series expansion of  $f(z) = 1/(2 - 3z + z^2)$  in the annulus  $\{z : \sqrt{2} < |z + i| < \sqrt{5}\}$ .
- 20% 5. Evaluate  $\int_{\sigma} (1 - z)^{-1} e^{1/z} dz$  for the cycle  $\sigma = \gamma + \beta$ , where  $\gamma(t) = 2e^{it}$  and  $\beta(t) = (-1/2) + e^{it}$  for  $0 \leq t \leq 2\pi$ .
- 10% 6. Evaluate the integral  $\int_0^{\infty} x(x^2 + 1)^{-2} \sin x dx$ .
- 20% 7. Given a polynomial  $P(z) = z^4 - 4z^3 + 11z^2 - 14z + 10$ , find which quadrants contain the zeros of  $P(z)$ .